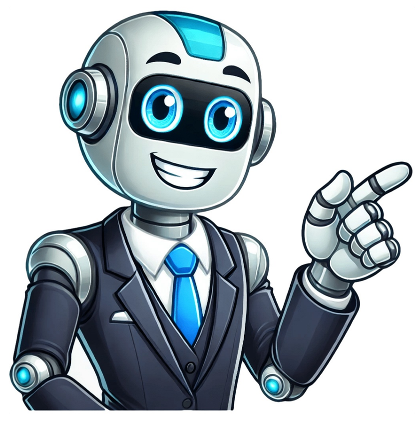

By the end of this section, you will be able to do the following: Describe the graphical method of vector addition and subtraction Use the graphical method of vector addition and subtraction to solve physics problems The learning objectives in this section will help your students master the following standards: (4) Science concepts. The student knows and applies the laws governing motion in two dimensions for a variety of situations. The student is expected to: (E) develop and interpret free-body force diagrams. Recall that a vector is a quantity that has magnitude and direction. For example, displacement, velocity, acceleration, and force are all vectors. In one-dimensional or straight-line motion, the direction of a vector can be given simply by a plus or minus sign. Motion that is forward, to the right, or upward is usually considered to be positive (+); and motion that is backward, to the left, or downward is usually considered to be negative (−). In two dimensions, a vector describes motion in two perpendicular directions, such as vertical and horizontal. For vertical and horizontal motion, each vector is made up of vertical and horizontal components. In a one-dimensional problem, one of the components simply has a value of zero. For two-dimensional vectors, we work with vectors by using a frame of reference such as a coordinate system. Just as with one-dimensional vectors, we graphically represent vectors with an arrow having a length proportional to the vector's magnitude and pointing in the direction that the vector points. [BL][OL]Review vectors and free body diagrams. Recall how velocity, displacement and acceleration vectors are represented. Figure 5.2 shows a graphical representation of a vector; the total displacement for a person walking in a city. The person first walks nine blocks east and then five blocks north. Her total displacement does not match her path to her final destination. The displacement simply connects her starting point with her ending point using a straight line, which is the shortest distance. We use the notation that a boldface symbol, such as D, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction is given by an angle represented by the symbol θ. θ. Note that her displacement would be the same if she had begun by first walking five blocks north and then walking nine blocks east. In this text, we represent a vector with a boldface variable. For example, we represent a force with the vector F, which has both magnitude and direction. The magnitude of the vector is represented by the variable in italics, F, and the direction of the variable is given by the angle θ. θ. Figure 5.2 A person walks nine blocks east and five blocks north. The displacement is 10.3 blocks at an angle 29.1 ∘ 29.1 ∘ north of east. The head-to-tail method is a graphical way to add vectors. The tail of the vector is the starting point of the vector, and the head (or tip) of a vector is the pointed end of the arrow. The following steps describe how to use the head-to-tail method for graphical vector addition. Let the x-axis represent the east-west direction. Using a ruler and protractor, draw an arrow to represent the first vector (nine blocks to the east), as shown in Figure 5.3(a). Figure 5.3 The diagram shows a vector with a magnitude of nine units and a direction of 0°. Let the y-axis represent the north-south direction. Draw an arrow to represent the second vector (five blocks to the north). Place the tail of the second vector at the head of the first vector, as shown in Figure 5.4(b). Figure 5.4 A vertical vector is added. If there are more than two vectors, continue to add the vectors head-to-tail as described in step 2. In this example, we have only two vectors, so we have finished placing arrows tip to tail. Draw an arrow from the tail of the first vector to the head of the last vector, as shown in Figure 5.5(c). This is the resultant, or the sum, of the vectors. Figure 5.5 The diagram shows the resultant vector, a ruler, and protractor. To find the magnitude of the resultant, measure its length with a ruler. When we deal with vectors analytically in the next section, the magnitude will be calculated by using the Pythagorean theorem. To find the direction of the resultant, use a protractor to measure the angle it makes with the reference direction (in this case, the x-axis). When we deal with vectors analytically in the next section, the direction will be calculated by using trigonometry to find the angle. [AL] Ask two students to demonstrate pushing a table from two different directions. Ask students what they feel the direction of resultant motion will be. How would they represent this graphically? Recall that a vector's magnitude is represented by the length of the arrow. Demonstrate the head-to-tail method of adding vectors, using the example given in the chapter. Ask students to practice this method of addition using a scale and a protractor. [BL][OL][AL] Ask students if anything changes by moving the vector from one place to another on a graph. How about the order of addition? Would that make a difference? Introduce negative of a vector and vector subtraction. This video shows four graphical representations of vector addition and matches them to the correct vector addition formula. There are two vectors a → and b → . The head of vector a → touches the tail of vector b → . The addition of vectors a → and b → gives a resultant vector c → . Can the addition of these two vectors can be represented by the following two equations? a → + b → = c → ; b → + a → = c → Yes, if we add the same two vectors in a different order it will still give the same resultant vector. No, the resultant vector will change if we add the same vectors in a different order. Vector subtraction is done in the same way as vector addition with one small change. We add the first vector to the negative of the vector that needs to be subtracted. A negative vector has the same magnitude as the original vector, but points in the opposite direction (as shown in Figure 5.6). Subtracting the vector B from the vector A, which is written as A − B, is the same as A + (−B). Since it does not matter in what order vectors are added, A − B is also equal to (−B) + A. This is true for scalars as well as vectors. For example, 5 – 2 = 5 + (−2) = (−2) + 5. Figure 5.6 The diagram shows a vector, B, and the negative of this vector, –B. Global angles are calculated in the counterclockwise direction. The clockwise direction is considered negative. For example, an angle of 30 ∘ 30 ∘ south of west is the same as the global angle 210 ∘ , 210 ∘ , which can also be expressed as −150 ∘ −150 ∘ from the positive x-axis. Now that we have the skills to work with vectors in two dimensions, we can apply vector addition to graphically determine the resultant vector, which represents the total force. Consider an example of force involving two ice skaters pushing a third as seen in Figure 5.7. Figure 5.7 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater. In problems where variables such as force are already known, the forces can be represented by making the length of the vectors proportional to the magnitudes of the forces. For this, you need to create a scale. For example, each centimeter of vector length could represent 50 N worth of force. Once you have the initial vectors drawn to scale, you can then use the head-to-tail method to draw the resultant vector. The length of the resultant can then be measured and converted back to the original units using the scale you created. You can tell by looking at the vectors in the free-body diagram in Figure 5.7 that the two skaters are pushing on the third skater with equal-magnitude forces, since the length of their force vectors are the same. Note, however, that the forces are not equal because they act in different directions. If, for example, each force had a magnitude of 400 N, then we would find the magnitude of the total external force acting on the third skater by finding the magnitude of the resultant vector. Since the forces act at a right angle to one another, we can use the Pythagorean theorem. For a triangle with sides a, b, and c, the Pythagorean theorem tells us that a 2 + b 2 = c 2 c= a 2 + b 2 . a 2 + b 2 = c 2 c= a 2 + b 2 . Applying this theorem to the triangle made by F1, F2, and Ftot in Figure 5.7, we get F tot 2 = F 1 2 + F 1 2 , F tot 2 = F 1 2 + F 1 2 , or F tot = (400 N) 2 + (400 N) 2 =566 N. F tot = (400 N) 2 + (400 N) 2 =566 N. Note that, if the vectors were not at a right angle to each other (90 ∘ (90 ∘ to one another), we would not be able to use the Pythagorean theorem to find the magnitude of the resultant vector. Another scenario where adding two-dimensional vectors is necessary is for velocity, where the direction may not be purely east-west or north-south, but some combination of these two directions. In the next section, we cover how to solve this type of problem analytically. For now let's consider the problem graphically. Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25 m in a direction 49 ∘ 49 ∘ north of east. Then, she walks 23 m heading 15 ∘ 15 ∘ north of east. Finally, she turns and walks 32 m in a direction 68 ∘ 68 ∘ south of east. Graphically represent each displacement vector with an arrow, labeling the first A, the second B, and the third C. Make the lengths proportional to the distance of the given displacement and orient the arrows as specified relative to an east-west line. Use the head-to-tail method outlined above to determine the magnitude and direction of the resultant displacement, which we'll call R. (1) Draw the three displacement vectors, creating a convenient scale (such as 1 cm of vector length on paper equals 1 m in the problem), as shown in Figure 5.8. Figure 5.8 The three displacement vectors are drawn first. (2) Place the vectors head to tail, making sure not to change their magnitude or direction, as shown in Figure 5.9. Figure 5.9 Next, the vectors are placed head to tail. (3) Draw the resultant vector R from the tail of the first vector to the head of the last vector, as shown in Figure 5.10. Figure 5.10 The resultant vector is drawn . (4) Use a ruler to measure the magnitude of R, remembering to convert back to the units of meters using the scale. Use a protractor to measure the direction of R. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since R is south of the eastward pointing axis (the x-axis), we flip the protractor upside down and measure the angle between the eastward axis and the vector, as illustrated in Figure 5.11. Figure 5.11 A ruler is used to measure the magnitude of R, and a protractor is used to measure the direction of R. In this case, the total displacement R has a magnitude of 50 m and points 7 ∘ 7 ∘ south of east. Using its magnitude and direction, this vector can be expressed as and θ= 7 ∘ south of east. θ= 7 ∘ south of east. The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that it does not matter in what order the vectors are added. Changing the order does not change the resultant. For example, we could add the vectors as shown in Figure 5.12, and we would still get the same solution. Figure 5.12 Vectors can be added in any order to get the same result. [BL][OL][AL] Ask three students to enact the situation shown in Figure 5.8. Recall how these forces can be represented in a free-body diagram. Giving values to these vectors, show how these can be added graphically. A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0 ∘ 66.0 ∘ north of east from her current location, and then travel 30.0 m in a direction 112 ∘ 112 ∘ north of east (or 22.0 ∘ 22.0 ∘ west of north). If the woman makes a mistake and travels in the opposite direction for the second leg of the trip, where will she end up? The two legs of the woman's trip are illustrated in Figure 5.13. Figure 5.13 In the diagram, the first leg of the trip is represented by vector A and the second leg is represented by vector B. We can represent the first leg of the trip with a vector A, and the second leg of the trip that she was supposed to take with a vector B. Since the woman mistakenly travels in the opposite direction for the second leg of the journey, the vector for second leg of the trip she actually takes is −B. Therefore, she will end up at a location A + (−B), or A − B. Note that −B has the same magnitude as B (30.0 m), but is in the opposite direction, 68 ∘ (180 ∘ − 112 ∘) 68 ∘ (180 ∘ − 112 ∘) south of east, as illustrated in Figure 5.14. Figure 5.14 Vector –B represents traveling in the opposite direction of vector B. We use graphical vector addition to find where the woman arrives A + (−B). (1) To determine the location at which the woman arrives by accident, draw vectors A and −B. (2) Place the vectors head to tail. (3) Draw the resultant vector R. (4) Use a ruler and protractor to measure the magnitude and direction of R. These steps are demonstrated in Figure 5.15. Figure 5.15 The vectors are placed head to tail. In this case and θ=7.5°south of east. θ=7.5°south of east. Because subtraction of a vector is the same as addition of the same vector with the opposite direction, the graphical method for subtracting vectors works the same as for adding vectors. A boat attempts to travel straight across a river at a speed of 3.8 m/s. The river current flows at a speed vriver of 6.1 m/s to the right. What is the total velocity and direction of the boat? You can represent each meter per second of velocity as one centimeter of vector length in your drawing. We start by choosing a coordinate system with its x-axis parallel to the velocity of the river. Because the boat is directed straight toward the other shore, its velocity is perpendicular to the velocity of the river. We draw the two vectors, vboat and vriver, as shown in Figure 5.16. Using the head-to-tail method, we draw the resulting total velocity vector from the tail of vboat to the head of vriver. Figure 5.16 A boat attempts to travel across a river. What is the total velocity and direction of the boat? By using a ruler, we find that the length of the resultant vector is 7.2 cm, which means that the magnitude of the total velocity is vtot=7.2m/s . vtot=7.2m/s . By using a protractor to measure the angle, we find θ= 32.0 ∘ . θ= 32.0 ∘ . If the velocity of the boat and river were equal, then the direction of the total velocity would have been 45°. However, since the velocity of the river is greater than that of the boat, the direction is less than 45° with respect to the shore, or x axis. Plot the way from the classroom to the cafeteria (or any two places in the school on the same level). Ask students to come up with approximate distances. Ask them to do a vector analysis of the path. What is the total distance travelled? What is the displacement? In this simulation, you will experiment with adding vectors graphically. Click and drag the red vectors from the Grab One basket onto the graph in the middle of the screen. These red vectors can be rotated, stretched, or repositioned by clicking and dragging with your mouse. Check the Show Sum box to display the resultant vector (in green), which is the sum of all of the red vectors placed on the graph. To remove a red vector, drag it to the trash or click the Clear All button if you wish to start over. Notice that, if you click on any of the vectors, the | R | | R | is its magnitude, θ θ is its direction with respect to the positive x-axis, Rx is its horizontal component, and Ry is its vertical component. You can check the resultant by lining up the vectors so that the head of the first vector touches the tail of the second. Continue until all of the vectors are aligned together head-to-tail. You will see that the resultant magnitude and angle is the same as the arrow drawn from the tail of the first vector to the head of the last vector. Rearrange the vectors in any order head-to-tail and compare. The resultant will always be the same. True or False—The more long, red vectors you put on the graph, rotated in any direction, the greater the magnitude of the resultant green vector. A vector is a quantity that has both magnitude and direction associated with it. This is different than a scalar quantity, which only corresponds to a magnitude. Velocity is an example of a vector quantity. It has both a magnitude (how fast something is going) and a direction (the direction it is traveling.) Vectors are often drawn as arrows. The length of the arrow corresponds to the magnitude of the vector, and the point of the arrow indicates the direction. There are two ways to work with vector addition and subtraction. The first is graphically, by manipulating the arrow diagrams of the vectors themselves. The second is mathematically, which gives exact results. When adding two vectors, you place the tail of the second vector to the tip of the first vector while maintaining vector orientation. The resultant vector is a vector that begins at the tail of the first vector and points in a straight line to the tip of the second vector. For example, consider adding vectors A and B which point in the same direction along a line. We place them "tip to tail" and the resultant vector, C, points in the same direction and has a length that is the sum of the lengths of A and B. Subtracting vectors in one dimension is essentially the same as adding except you "flip" the second vector. This results directly from the fact that subtraction is the same as adding a negative. When working in one dimension, the direction of a vector can be indicated by sign. We choose one direction to be the positive direction (typically "up" or "right" are chosen as positive), and assign any vector pointing in that direction as a positive quantity. Any vector pointing in the negative direction is a negative quantity. When adding or subtracting vectors, add or subtract their magnitudes with the appropriate signs attached. Suppose in the previous section, vector A had a magnitude of 3 and vector B had a magnitude of 5. Then resultant vector C = A + B = 8, a vector of magnitude 8 pointing in the positive direction, and resultant vector D = A – B = -2, a vector of magnitude 2 pointing in the negative direction. Note that this is consistent with the graphical results from before. Tip: Be careful to only add vectors of the same type: velocity + velocity, force + force and so on. As with all math in physics, the units must match up! If the first vector and the second vector are not along the same line in Cartesian space, you can use the same "tip to tail" method to add or subtract them. To add two vectors, simply imagine lifting the second one and placing its tail to the tip of the first while maintaining its orientation as shown. The resultant vector is an arrow beginning at the tail of the first vector and ending at the tip of the second vector: Just as in one dimension, subtracting one vector from another is equivalent to flipping and adding. Graphically, this looks like the following: Note: Sometimes vector addition is shown graphically by putting the tails of the two addend vectors together and creating a parallelogram. The resultant vector is then the diagonal of this parallelogram. To add and subtract vectors in two dimensions mathematically, follow these steps: Decompose each vector into an x-component, sometimes called the horizontal component, and a y-component, sometimes called the vertical component, using trigonometry. (Note that components may be either negative or positive depending on which direction the vector is pointing) Add the x-components of both vectors together, and then add the y-components of both vectors together. This result gives you the x and y components of the resultant vector. The magnitude of the resultant vector can be found using the Pythagorean theorem. The direction of the resultant vector can be found via trigonometry using the inverse tangent function. This direction is typically given as an angle with respect to the positive x-axis. Recall the relationships between the sides and angles of a right triangle from trigonometry. \(\sin(\theta)=\frac{b}{c}\text{ }\ \cos(\theta)=\frac{a}{c} \text{ }\ \tan(\theta)=\frac{b}{a}\) Pythagorean theorem: \(c^2=a^2+b^2\) Projectile motion provides classic examples of how we might use these relationships to both decompose a vector and determine the final magnitude and direction of a vector. Consider two people playing catch. Suppose you are told the ball is thrown from a height of 1.3 m with a speed of 16 m/s at an angle of 50 degrees with the horizontal. In order to begin analyzing this problem, you will need to decompose this initial velocity vector into x and y components as shown: \(v_{xi}=v_i\cos(\theta)=16\times\cos(50)=10.3 \text{ m/s}\ v_{yi}=v_i\sin(\theta)=16\times\sin(50)=12.3\text{ m/s}\) If the catcher misses the ball and it hits the ground, with what final velocity will it strike? Using kinematic equations, we are able to determine that the final components of the ball's velocity are: \(v_{xf}=10.3 \text{ m/s}\ v_{yf}=-13.3\text{ m/s}\) The Pythagorean theorem allows us to find the magnitude: \(v_{f}=\sqrt{(10.3)^2+ (-13.3)^2}=16.8\text{ m/s}\) And trigonometry allows us to determine the angle: \(\theta=\tan^{-1}\Big(\frac{-13.3}{10.3}\Big)=-52.2\degree\) Consider a car rounding a corner. Suppose _vi_ for the car is in the x-direction with magnitude 10 m/s, and _vf_ is at a 45-degree angle with the positive x-axis with magnitude 10 m/s. If this change in motion occurs in 3 seconds, what is the magnitude and direction of the car's acceleration as it turns? Recall that acceleration is a is a vector quantity defined as: \(a=\frac{(v_f-v_i)}{t}\) Where _vf_ and _vi_ are final and initial velocities respectively (and hence, are also vector quantities). In order to compute the vector difference _**vf** - **vi**,_ we must first decompose the initial and final velocity vectors: \(v_{xi}=10\text{ m/s}\ v_{yi}=0\text{ m/s}\ v_{xf}=10\cos(45)=7.07\text{ m/s}\ v_{yf}=10\sin(45)=7.07\text{ m/s}\) Then we subtract the final x and y components from the initial x and y components to get components of _**vf** - **vi**_: Then we subtract the x and y components: \((v_f-v_i)_x=v_{xf}-v_{xi}=7.07-10=-2.93\text{ m/s}\ (v_f-v_i)_y=v_{yf}-v_{yi}=7.07-0=7.07\text{ m/s}\) Then divide each by time to get the components of the acceleration vector: \(a_x=\frac{-2.93}{3}=-0.977\text{ m/s}^2\text{ }\ a_y=\frac{7.07}{3}=2.36\text{ m/s}^2\) Use the Pythagorean theorem to find the magnitude of the acceleration vector: \(a=\sqrt{(-0.977)^2+(2.36)^2}=2.55\text{ m/s}^2\) Finally, use trigonometry to find the direction of the acceleration vector: \(\theta=\tan^{-1}\Big(\frac{2.36}{-0.977}\Big)=113\degree\) Wolfram Mathworld: Vector Addition The Physics Classroom: Vector Addition TOWELL, GAYLE. "How To Add & Subtract Vectors (W/ Diagrams)" sciencing.com, . 28 December 2020. APA TOWELL, GAYLE. (2020, December 28). How To Add & Subtract Vectors (W/ Diagrams). sciencing.com. Retrieved from Chicago TOWELL, GAYLE. How To Add & Subtract Vectors (W/ Diagrams) last modified March 24, 2022. Download Article Download Article Many common physical quantities are often vectors or scalars. Vectors are akin to arrows and consist of a positive magnitude (length) and importantly a direction. on the other hand scalars are just numerical values sometimes possibly negative. Note that although vector magnitudes are positive or perhaps zero the components of vectors can of course be negative indicating vector directed contrary to the coordinate or reference direction. Examples of vectors: force, velocity, acceleration, displacement, weight, magnetic field, etc. Examples of scalars: mass, temperature, speed, distance, energy, voltage, electric charge, pressure within a fluid, etc. While scalars can be added directly like numbers (e.g. 5 kJ of work plus 6kJ equals 11kJ ; or 9 volt plus minus 3 volt gives 6 volt: +9v plus -3v gives +6v), vectors are slightly more complicated to add or subtract, although collinear vectors are easy and behave like adding numbers which may be negative. See below several ways to tackle vector addition and subtraction. Add two vectors by adding together each vector's matching components. Similarly, subtract vectors by subtracting each of their matching components. To add vectors visually, you'll need to draw them from head to tail so that the second vector's tail meets the first's vector's head. When subtracting vectors visually, reverse the direction of the vector but keep its magnitude the same. 1 Express a vector in terms of components in some coordinate system usually x, y, and possibly z in usual 2 or 3 dimensional space (higher dimensionality is possible too in some mathematical situations). These component parts are usually expressed with a notation similar to that used to describe points in a coordinate system (e.g. , etc.). If these pieces are known, adding or subtracting vectors is just a simple adding or subtracting the x, y, and z components.[1] Note that vectors can be 1, 2, or 3-dimensional. Thus, vectors can have an x component, an x and y component, or an x, y, and z component. Let's say that we have two 3-dimensional vectors, vector A and vector B. We might write these vectors in components as A = and B = , using x y z components accordingly. 2 To add two vectors, we simply add their components. In other words, add the x component of the first vector to the x component of the second and so on for y and z. The answers you get from adding the x, y, and z components of your original vectors are the x, y, and z components of your new vector.[2] In general terms, A+B = . Let's add two vectors A and B. Example: A = and B = . A + B = , or . Advertisement 3 To subtract two vectors, subtract their components. Note that subtracting one vector from another A-B can be thought of adding the "reverse" of that second A+(-B).[3] In general terms, A-B = Let's subtract two vectors A and B. A = and B = . A - B = , or . Advertisement 1 Represent vectors visually by drawing them with a head and tail. Since vectors have magnitude and direction, they are likened to arrows with a tail and a head and a length. Vectors can be said to have a "beginning point" and an "end point". The "sharp point" of the arrow is the vector's head and the "base" of the arrow is the tail.[4] When making a scale drawing of a vector, you must take care to measure and draw all angles accurately. Mis-drawn angles will lead to poor answers. 2 To add 2 vectors, draw the second vector B so that its tail meets the head of the first A. This is referred to as joining your vectors "head to tail". If you are only adding two vectors, this is all you'll need to do before finding your resultant vector A+B. Vector B may need to be slid into position without altering its orientation, called parallel transport.[5] Note that the order you join the vectors in is not important. Vector A + Vector B = Vector B + Vector A 3 To subtract, add the "negative" of the vector. Subtracting vectors visually is fairly simple. Simply reverse the vector's direction but keep its magnitude the same and add it to your vector head to tail as you would normally. In other words, to subtract a vector, turn the vector 180o around and add it.[6] 4 If adding or subtracting more than two vectors, join all other vectors head-to-tail in sequence. Actually the order in which you join the vectors does not matter. This method can be used for any number of vectors.[7] 5 To get the result: Draw a new vector from tail of the first vector to the head of the last. Whether you are adding/subtracting two vectors or a hundred, the vector stretching from the original starting point (the tail of your first vector) to end point of your final added vector (the head of your last vector) is the resultant vector, or the sum of all your vectors.[8] Note that this vector is identical to the vector obtained by adding the x,y, and perhaps z components of all the vectors separately. If you drew all of your vectors to scale, measuring all angles exactly, you can find the magnitude of the resultant vector by measuring its length. You can also measure the angle that the resultant makes with either a specified vector or the horizontal/vertical etc. to find its direction. If you didn't draw all vectors to scale, you probably need to calculate the magnitude of the resultant using trigonometry. You may find the Sine Rule and the Cosine Rule helpful here.[9] If you are adding more than two vectors together, it is helpful to first add two, then add their resultant with the third vector, and so on. See the following section for more information. 6 Represent your resultant vector via its magnitude and direction.[10] Vectors are defined by their length and direction. As noted above, assuming you drew your vectors accurately, your new vector's magnitude is its length and its direction is its angle relative to the vertical, horizontal, etc. Use the units of your added or subtracted vectors to choose the units for your resultant vector's magnitude.[11] For example, if the vectors we added represented velocities in ms-1, we might define our resultant vector as "a velocity of x ms-1 at yo to the horizontal". Advertisement 1 Use trigonometry to find a vector's components. To find a vector's components, it's usually necessary to know its magnitude and its direction relative to the horizontal or vertical and to have a working knowledge of trigonometry. Taking a 2-D vector first: set or imagine your vector as the hypotenuse of a right triangle whose other two sides are parallel to the x and y axes. These two sides can be thought of as head-to-tail component vectors that add to create your original vector.[12] The lengths of the two sides are equal to the magnitudes of the x and y components of your vector and may be calculated using trigonometry.[13] If x is the magnitude of the vector, the side adjacent to the vector's angle (relative to the horizontal, vertical, etc.) angle is xcos(θ), while the side opposite is xsin(θ). It's also important to note the direction of your components. If the component points in the negative direction of one of your axes, it is given a negative sign. For example, in a 2-D plane, if a component points to the left or downwards, it is given a negative sign. For example, let's say that we have a vector with a magnitude of 3 and a direction of 135o relative to the horizontal. With this information, we can determine that its x component is 3cos(135) = -2.12 and its y component is 3sin(135) = 2.12 2 Add or subtract two or more vectors' corresponding components.[14] When you've found the components of all of your vectors, simply add their magnitudes together to find the components of your resultant vector. First, add all the magnitudes of the horizontal components (those parallel to the x-axis) together. Separately, add all the magnitudes of the vertical components (those parallel to the y-axis). If a component has a negative sign (-), its magnitude is subtracted, rather than added. The answers you obtain are the components of your resultant vector. For instance, let's say that our vector from the previous step, , is being added to the vector . In this case, our resultant vector would be , or . 3 Calculate the magnitude of the resultant vector using the Pythagorean Theorem.[15] The Pythagorean Theorem, c2=a2+b2, solves for the side lengths of right triangles. Since the triangle formed by our resultant vector and its components is a right triangle, we can use it to find our vector's length and therefore its magnitude. With c as the magnitude of the resultant vector, which you're solving for, set a as the magnitude of its x component and b as the magnitude of its y components. Solve with algebra. To find the magnitude of the vector whose components we found in the previous step, , let's use the Pythagorean Theorem. Solve as follows: c2=(3.66)2+(-6.88)2 c2=13.40+47.33 c=√60.73 = 7.79 4 Calculate the direction of the resultant with the tangent function.[16] Finally, find the resultant vector's direction. Use the formula θ=tan-1(b/a), where θ is the angle that the resultant makes with the x-axis or the horizontal, b is the magnitude of the y component, and a is the magnitude of the x component. To find the direction of our example vector, let's use θ=tan-1(b/a). θ=tan-1(-6.88/3.66) θ=tan-1(-1.88) θ=-61.99o 5 Represent your resultant vector via its magnitude and direction.[17] As noted above, vectors are defined by their magnitude and direction. Be sure to use the proper units for your vector's magnitude. For example, if our example vector represented a force (in Newtons), then we might write it as "a force of 7.79 N at -61.99o to the horizontal". Advertisement Add New Question Question How can I find the resultant if angles are not given and only magnitudes are given? If these are vectors, and you have no other information about their direction, you can't! Since you don't know the angles (or the relative alignment) between them, it's possible that the vectors could line up exactly (in which case the resultant has a magnitude equal to the sum of their magnitudes), or they could point in opposite directions (in which case the resultant has a magnitude equal to the difference between their magnitudes), or anywhere in between. If you were given a problem like this, it is not completely specified. Question How do I add six or more than six vectors? Adding n vectors is easy because vectors obey the superposition principle. Simply add their components. Question I thought one of the processes in adding vectors is cross-multiplication? The cross product is a type of multiplication, not addition, which pertains to this article. See more answers Ask a Question Advertisement Thanks Thanks Thanks Show More Tips Advertisement This article was reviewed by Joseph Meyer. Joseph Meyer is a High School Math Teacher based in Pittsburgh, Pennsylvania. He is an educator at City Charter High School, where he has been teaching for over 7 years. Joseph is also the founder of Sandbox Math, an online learning community dedicated to helping students succeed in Algebra. His site is set apart by its focus on fostering genuine comprehension through step-by-step understanding (instead of just getting the correct final answer), enabling learners to identify and overcome misunderstandings and confidently take on any test they face. He received his MA in Physics from Case Western Reserve University and his BA in Physics from Baldwin Wallace University. This article has been viewed 614,437 times. Co-authors: 43 Updated: January 1, 2025 Views: 614,437 Categories: Coordinate Geometry | Linear Algebra Print Send fan mail to authors Thanks to all authors for creating a page that has been read 614,437 times. "The pictures are more than helpful, very clearly getting the point across with exactly the information you need." Share your story We can do vector subtraction just like how we do the subtraction of scalars. We subtract the corresponding components of vectors while subtracting vectors. The graphical interpretation of vector subtraction can be understood by using the parallelogram law and triangle law of addition. Let us learn more about vector subtraction along with geometrical interpretation and examples. What is Vector Subtraction? The vector subtraction of two vectors a and b is represented by a - b and it is nothing but adding the negative of vector b to the vector a. i.e., a - b = a + (-b). Thus, subtraction of vectors involves the addition of vectors and the negative of a vector. The result of vector subtraction is again a vector. The following are the rules for subtracting vectors: It should be performed between two vectors only (not between one vector and one scalar). Both vectors in the subtraction should represent the same physical quantity. Let us understand how to subtract vectors graphically in the upcoming sections. Vector Subtraction by Parallelogram Law Suppose that a and b are two vectors. How can we interpret the subtraction of these vectors graphically? That is, what meaning do we attach to a - b? To start with, we note that a - b will be a vector which when added to b should give back a. i.e., (a - b) + b = a But how do we determine the vector a - b, given the vectors a and b? The following figure shows vectors a and b (we have drawn them to be co-initial). Using the parallelogram law of vector addition, we can determine the vector as follows. We interpret a - b as a + (- b), that is, the vector sum of a and −b. Now, we reverse vector b, and then add a and -b using the parallelogram law: This shows the vector subtraction a - b as the addition of a and −b. Vector Subtraction by Triangle Law Now, we will interpret the subtraction of vectors using the triangle law of vector addition. Denote the vector drawn from the end-point of b to the end-point of a by c. Note that b + c = a. Thus, c = a - b. In other words, the vector a - b is the vector drawn from the tip of b to the tip of a (if a and b are co-initial). Note that both ways (using parallelogram law and triangle law) are described above give us the same vector for a - b. This becomes clearer from the figure below: The vector PT is obtained by adding a and −b using the parallelogram law. The vector RQ is obtained by drawing the vector from the tip of b to the tip of -a. Clearly, both vectors are the same (as their magnitudes and directions are the same). How to Subtract Vectors? Here are multiple ways of subtracting vectors: To subtract two vectors a and b graphically (i.e., to find a - b), just make them coinitial first and then draw a vector from the tip of b to the tip of a. We can add -b (the negative of vector b which is obtained by multiplying b with -1) to a to perform the vector subtraction a - b. i.e., a - b = a + (-b). If the vectors are in the component form we can just subtract their respective components in the order of subtraction of vectors. Here is an example. Example: If a = and b = then find a - b. Solution: a - b = - = = Therefore, a - b = . Properties of Vector Subtraction Here are some important properties of vector subtraction. Any vector subtracted from itself results in a zero vector. i.e., a - a = 0, for any vector a. The subtraction of vectors is NOT commutative. i.e., a - b is not necessarily equal to b - a. The vector subtraction is NOT associative. i.e., (a - b) - c does not need to be equal to a - (b - c). (a - b) · (a + b) = |a|2 - |b|2. (a - b) · (a - b) = |a - b|2 = |a|2 + |b|2 - 2 a · b. ☛ Related Topics: Example 1: Compute the vector subtraction a - b if a = and b = . Also, find its magnitude. Solution: Given that a = and b = . Now we will find their difference by subtracting the respective components. a - b = - = Its magnitude is, |a - b| = √[(-2)2 + (-1)2 + 32] = √(4 + 1 + 9) = √14 Answer: a - b = and its magnitude is √14. Example 2: If θ is the angle between two unit vectors a and b, then find the magnitude of their difference in terms of θ. Solution: From the properties of vector subtraction, |a - b|2 = |a|2 + |b|2 - 2 a · b ... (1) By the definition of dot product, a · b = |a| |b| cos θ Also, since a and b are unit vectors, |a| = |b| = 1. Substituting these in (1): |a - b|2 = |a|2 + |b|2 - 2 |a| |b| cos θ = 1 + 1 - 2 (1)(1) cos θ = 2 - 2 cos θ = 2 (1 - cos θ) Using trig formulas, 1 - cos θ = 2 sin2 (θ/2). So |a - b|2 = 2 (2 sin2(θ/2)) = 4 sin2(θ/2). Taking square root, |a - b| = 2 sin (θ/2). Answer: |a - b| = 2 sin (θ/2). Example 3: What can you say about two vectors a and b if the magnitudes of their sum and difference are equal to each other? Solution: It is given that |a - b| = |a + b| Squaring on both sides, |a - b|2 = |a + b|2 By the properties of vector subtraction, |a|2 + |b|2 - 2 |a| |b| cos θ = |a|2 + |b|2 + 2 |a| |b| cos θ 4|a| |b| cos θ = 0 cos θ = 0 θ = π/2 Answer: a and b are perpendicular. View Answer > go to slidego to slidego to slide Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts. Book a Free Trial Class FAQs on Vector Subtraction Vector subtraction of two vectors a and b is just the addition of vectors a and -b. i.e., a - b = a + (-b). To find -b, we have to multiply each component of vector b by -1. How Do You Subtract Vectors? To subtract a vector b from another vector a, just make them coinitial and then draw a vector from the tip of b to the tip of a. To subtract a vector b from another vector a when their components are given, then just subtract every component of b from the corresponding component of a. How to Find Vector Difference Graphically? To find the vector difference of two vectors a and b graphically: Draw both vectors to start from the same initial point. Draw the difference vector a - b from the tip of b to the tip of a. How to Subtract Vectors in Component Form? If two vectors are a = and b = , then their difference is represented by a - b and is obtained by subtracting components of b from the respective components of a. i.e., a - b = . What is the Parallelogram Law of Vector Subtraction? The parallelogram law of subtraction says "if two vectors a and -b are starting from a point P and they are two adjacent sides of a parallelogram, then their sum which is a + (-b) (which can also be written as a - b) is the vector that represents the diagonal of a parallelogram that starts from P". What is the Triangle Law of Vector Subtraction? The triangle law of vector subtraction says "to find the difference a - b of two vectors a and b (that are coinitial), just draw a vector from the tail of b to the tail of a". Last updated on April 16th, 2021 at 10:04 am As we have discussed Vector in physics and had a detailed study of Vector addition, now its turn to discuss another topic of vector physics which is Vector Subtraction. Vector Subtraction helps us to subtract one vector from another vector. Now how do it? To subtract one vector from another, for example, to get A – B, simply form the vector –B, which is the scalar multiple (–1)B, and add it to A i.e.A – B = A + (–B) Example: For the two vectors A and B in the diagram 1, find the vector A– B. Diagram 1 Solution. Flip B around i.e. reverse the direction of B (thereby forming –B) and add that vector to A using triangle law of vector addition. Diagram 2 Here is another example. The diagram 3 below shows 2 vectors A and B and how A-B is derived. Diagram 3 Now let's discuss one use case of vector subtraction where we need to subtract a vector from another vector. If you are aware of the term 'relative velocity', probably you have already got a hint. When we discuss the motion of different objects situated near the earth's surface, we take earth surface as the 'static' frame of reference. An object is said to be in motion with respect to a viewer on this static frame of reference.That's cool. But if we have to consider the motion of an object with respect to a 'moving frame of reference', then how to handle that situation? For example, to a passenger in a running train, all nearby houses, trees, and other static objects seem to be in motion. That means with respect to that passenger in a running train those objects have a 'relative velocity', though they are actually motionless or static. Relative velocity calculation – When we have to calculate the relative velocity of an object with respect to a viewer in motion, we have to subtract the actual velocity of the viewer from the actual velocity of the object. Say the actual velocity of the object is V1 and the actual velocity of the viewer is V2. Then the relative velocity of the object with respect to the viewer in motion =Vr = V1 – V2 Now while calculating the difference between V1 and V2, we will use the vector addition process but with a tweak. It's here: V1-V2 = V1 + (-V2) So we have to consider the negative of the V2 vector, which means a vector with the same magnitude as V2 but with reverse direction. And then follow the known vector addition process between V1 and (-V2) and get the vector V1-V2. 1)A car is moving with a velocity of 80 km/hr towards north. A bus is moving with a velocity of 80√2 km/hr towards north-west. What is the relative velocity of the bus with respect to the car? 2)One ship is moving towards east with a velocity of 10 kmph. A boat is moving towards north-east, making an angle 30 degree with north. If to a viewer on the ship the boat seems to move towards north, then what is the velocity (magnitude) of the boat? With the help of vector subtraction mechanism we just discussed, we can easily find out the relative velocity of rain with respect to a moving observer.If there is not any considerable wind flow then rain drops come down vertically. Now a person at rest can hold the umbrella vertically above his head and can escape the raindrops. But if the person himself is in motion then he has to hold the umbrella in a slanted position pointing towards his front side to save from rain. We can take help of vector diagram and vector subtraction procedure to discuss this. Say, the vertically downward velocity of raindrops is V and the person is moving towards east with a velocity U. Now say, with respect to the moving observer (the person) the relative velocity of the raindrops is R. So we will get R by subtracting observer's velocity from the rain velocity. R = V – U = V + (-U) In the diagram, a reverse vector of U is drawn by flipping U and is shown as –U. Remember that this flipped vector has same magnitude as that of the original U but the – sign only shows a reverse direction. In the following calculation, we will use its magnitude only, as we can take it as a new vector. Now if we add these 2 vectors V and –U using Vector addition (Here Pythagoras theorem is a better option to do this addition as the 2 vectors V and –U are at right angle), we get R, the relative velocity of rain. Magnitude of R = √(V2 + U2) ……………….(1) And also we get the angle θ it makes with vertical line, from the equation below: tan θ = U/V …………….(2) As we note that R makes an angle θ with vertical, so it's evident that the raindrops seem to come in a slanted path from the front side of the person in motion. See also Scalar product formula with an example A vector can be defined as a quantity, measurement, or object that has both a magnitude and a direction. It is one of the most important and basic concepts in Physics that finds its applications in almost all the branches of the subject. A vector can be visualised geometrically as a directed line segment. The length of the line segment corresponds to the magnitude of the vector and the arrowhead corresponds to the direction of the vector. The direction of any vector is defined from its tail to its head where the arrow is there.Most common examples of vector quantities include force, velocity, acceleration, displacement, momentum, electric field, etc. Basic algebraic operations can be applied to vectors, but they have their own rules for these operations. We cannot add or subtract vectors like we add or subtract numbers. Vectors have directions too, so this has to be taken into consideration. We will look at how to subtract vectors in detail in this article. Image: Geometrical representation of a Vector What is Subtraction of Vectors?Suppose there are vectors a and b. The subtraction of these vectors is represented as, a-b. In simple words, it is just the addition of the vector -b to the vector a. It can be written as, a-b=a+(-b)One can visualise vector subtraction as the addition of a vector to another vector after rotating it 180° in space. Subtraction of vectors involves the addition of vectors and the negative of any vector. It is obvious that the subtraction of two vectors will give a vector as the result. Two rules can be laid out for vector subtraction.Vector subtraction can be performed only between two vectors.The vectors which are being subtracted should represent the same physical quantity, otherwise, they cannot be subtracted. Parallelogram Law of Vector SubtractionLet's continue with the same vectors a and b. These vectors can be visualised in space as given in the diagram given below. To apply the parallelogram law the vectors have to be coterminal or their initial points should coincide. Image: Parallelogram Law of Vector Subtraction The basic question before starting subtraction is how to understand what the subtraction of these vectors means. The subtraction is a-b and when b is added to this subtraction, the answer should be a. This can be shown as,(a-b)+b=aThe above figure shows vectors a and b. Now subtraction basically means adding -b to a. If we rotate the vector b by 180° and then add it to a, we'll have our answer. The figure below helps visualise this process. Now forming a parallelogram by forming lines around the two vectors will give a parallelogram as shown in the above figure. Then a diagonal drawn from the initial point of the vectors to the opposite vertex will give the resultant, in this case, the subtraction. So we have the resultant, a-b by using the parallelogram law. Triangle Law of Vector SubtractionTo use the triangle law of vector subtraction, the vectors don't need to have the same initial point. The initial point of one vector needs to be at the terminal point of the previous vector. Then the resultant of the vectors, or the addition of those vectors will be drawn from the initial point of the first vector to the terminal point of the last vector. For subtraction, the second vector should be rotated by 180° and then the triangle law can be applied to get the resultant or the subtraction of those vectors. It can be visualised by using the figure below. Image: Triangle Law for Vector Subtraction Here the vector b was rotated by 180° and then the resultant was drawn from the initial point of a to the tip of -b to get a-b.Vector Subtraction FormulaLet's continue with vectors a and b. Apart from the graphical ways of subtracting the vectors, they can also be subtracted by subtracting their respective components from each other. Let,a={a1, a2} b={b1, b2}The respective components can be subtracted to get a-b.a-b=a+(-b) a-b={(a1-b1), (a2-b2)}This is the vector subtraction formula. This can be extended to any number of components that the vector has.Let's take a vector subtraction example to understand things better. Suppose a={3,5}, and b={2,6}.The subtraction will be given as,a-b={(3-2), (5-6)} a-b={1, -1}It is important to know that the subtraction of vectors obeys which laws. Subtraction of vectors obeys the distributive law, that is a(b-c)=ab-acHere a, b, and c are vectors.ConclusionThe subtraction of vectors is the same as adding the reverse or inverse of one vector to another vector. The parallelogram law of subtraction can be used to calculate the subtraction of the vectors by drawing a parallelogram taking the two vectors with the same initial point. Then the resultant can be drawn as a diagonal from the initial point to the opposite vertex. Similarly, the triangle law can also be used to find the subtraction of vectors. The vectors have to be arranged by keeping the initial point of the second vector on the tip of the first vector and then drawing the resultant from the initial point of the first vector to the tip of the second vector. Also, the individual components can be subtracted to get the subtraction. What is a vector? A vector is a quantity that has magnitude (size) and direction. How to represent a vector graphically, in column-vector form and in unit-vector form? Show Step-by-step Explanations Vector subtraction is the operation of finding the difference between two vectors. Subtracting a vector v from a vector u (written as u - v) is equivalent to adding the negative of vector v to vector u (u + (-v)). The following diagram shows how to subtract vectors graphically. Scroll down the page for more examples and solutions for vector subtraction. Subtracting a vector is the same as adding its negative. The negative of a vector has the same magnitude (length) as the original vector but points in the opposite direction. The difference of the vectors p and q is the sum of p and -q. p – q = p + (-q) Geometric Method (Head-to-Tail or Triangle Law): Draw the first vector p starting from an initial point. Draw the negative of the second vector -q starting from the tail (arrow tip) of the first vector p. Remember to reverse the direction of -q while keeping its length the same. The resultant vector p - q is the vector drawn from the head of the first vector p to the tail of the negative of the second vector -q. Example: Subtract the vector v from the vector u.   Show Solution u – v = u + (-v) Change the direction of vector v to get the vector -v. Check: The column vector should represent the vector that was drawn. How to subtract vectors using column vectors? u - v = u + (-v) Since we know how to add vectors and multiply by negative one, we can also subtract vectors. Show Video Lesson How to subtract vectors using column vectors and explained graphically? Show Video Lesson How to subtract vectors graphically? Show Video Lesson Geometric subtraction of two vectors Show Video Lesson How to solve word problems using vector subtraction? Vector word problems when given magnitude and direction Subtract the following vectors (B - A) A = 5.0 m at 40 degrees west of North B = 2.5 m south. Find the distance and direction of (B - A) Show Video Lesson Vector subtraction including boat example Introduction to 'head to tail' vector subtraction in the geometric sense. This is then applied to an example of working out a boat's velocity relative to water given the velocity of the current and the velocity of the boat relative to land are both known. velocity relative to land = water velocity + boat's velocity relative to water. Example: A woman wants to get to a destination that is due North of her starting point. To do this, she needs to row across a stream. The current is flowing East at 12km/hr. The woman can row the boat at a constant speed of 16km/hr. What will be the required direction that she must row the boat in order to reach the required destination? 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