Addition: a + b = b + a Multiplication: a × b = b × a Addition: (a + b) + c = a + (b + c) Multiplication: (a × b) × c = a × (b × c) a × (b + c) = (a × b) + (a × c) Addition: a + 0 = a Multiplication: a × 1 = a Addition: a + (-a) = 0 Multiplication: a × (1/a) = 1 1. Order of Operations (PEMDAS/BODMAS) Brackets/Parentheses Orders (Exponents/Powers) Multiplication and Division (left to right) Addition and Subtraction (left to right) Things equal to the same thing are equal to each other 3. Properties of Zero Any number plus zero equals the number Any number times zero equals zero 4. Properties of One Any number times one equals the number Any number divided by one equals the number 5. Sign Rules Positive × Positive = Positive Negative × Negative = Positive Positive × Negative = Negative Mathematical statements must be consistent across all operations Equal values can be substituted for each other in any expression Addition Rule When adding positive numbers, the sum increases When adding negative numbers, the sum decreases Multiplication Rule Positive × Positive = Positive Negative × Negative = Positive Positive × Negative = Negative The standard rule refers to the order of operations: Brackets/Parentheses Orders (Exponents/Powers) Division and Multiplication (left to right) Addition and Subtraction (left to right) Rule 5 refers to the fifth step in problem-solving: Read the problem Understand the given information Determine what's being asked Plan the solution Execute the plan and check the answer Rule 3 refers to the distributive property: a(b + c) = ab + ac If a = b and b = c , then a = c If a = b , then a + c = b + c If a = b , then a × c = b × c Counting Rules Numbers go from left to right Each number has its own symbol Zero means nothing Addition Rules Adding makes numbers bigger Order doesn't matter (2+3 = 3+2) Subtraction Rules Subtraction makes numbers smaller Order matters (5-3 ≠ 3-5) What you do to one side of an equation, you must do to the other side to maintain equality. Two quantities that are equal remain equal when the same operation is performed on both. Algebra problems are easier to solve when you know the rules and formulas. Memorizing key algebra formulas will speed up your work, too. And if you know the rules of divisibility and the order of operations, you'll be able to solve algebra problems without getting stressed. Some formulas occur frequently when you're doing algebraic manipulations and working through mathematical applications. You'll find ways to use these algebra formulas even when you're doing something other than algebra, such as planning a garden or a road trip. Of course, you need to know what the letters and symbols in the formula mean, so both the formulas and the explanations are in this table. In algebra, knowing the rules of divisibility can help you solve faster. When factoring algebraic expressions to solve equations, you need to be able to pull out the greatest factor. You also need common factors when reducing algebraic fractions. The rules of divisibility help you find the common factors and change the algebraic expressions so that they're put in a more workable form. Divisibility by 2: A number is divisible by 2 if the last digit in the number is 0, 2, 4, 6, or 8. Divisibility by 3: A number is divisible by 3 if the sum of the digits in the number is divisible by 3. Divisibility by 4: A number is divisible by 4 if the last two digits in the number form a number divisible by 4. Divisibility by 5: A number is divisible by 5 if the last digit is 0 or 5. Divisibility by 6: A number is divisible by 6 if it is divisible by both 2 and 3. Divisibility by 8: A number is divisible by 8 if the last three digits form a number divisible by 8. Divisibility by 9: A number is divisible by 9 if the sum of the digits of the number is divisible by 9. Divisibility by 10: A number is divisible by 10 if it ends in 0. Divisibility by 11: A number is divisible by 11 if the sums of the alternate digits are different by 0, 11, 22, or 33, or any two-digit multiple of 11. In other words, say you have a six-digit number: Add up the first, third, and fifth digits — the odd ones. Then add the digits in the even places — second, fourth, and sixth. Then subtract the smaller of those totals from the larger total, and if the answer is a multiple of 11, the original number is divisible by 11. Divisibility by 12: A number is divisible by 12 if the last two digits form a number divisible by 4 and if the sum of the digits is divisible by 3. Solve algebra problems correctly by following the order of operations. When performing more than one operation on an algebraic expression, work out the operations and signs in the following order: First calculate powers and roots. Then perform all multiplication and division. Finish with addition and subtraction. If you have more than two operations of the same level, do them in order from left to right, following the order of operations. For example, to solve 24 ÷ 3 + 11 – 32 × 2, you would First calculate powers and roots. This problem doesn't have any roots, but it does have one power, 32. You know that 32 = 3 × 3 = 9. Substitute 9 into the problem, and you get 24 ÷ 3 + 11 – 9 × 2. Then perform all multiplication and division. Working left to right, 24 ÷ 3 = 8 and 9 × 2 = 18. Substitute those numbers into the problem, and you have 8 + 11 – 18. Finish with addition and subtraction. So, you end up with 8 + 11 – 18 = 19 – 18 = 1. Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Algebra is the field of mathematics which deals with representation of a situation using mathematical symbols, variables and arithmetic operations like addition, subtraction, multiplication and division leading to the formation of relevant mathematical expressions. In this lesson we will go through all the rules of algebra, operations and formulas. Algebra Basics We need to know the basic terminology which relates to algebra in order to understand its basics. An expression consisting of 4 main parts, variables, operators, exponents, coefficients and constants along with an equal to symbol is known as an algebraic equation. Let us take an equation, ax2 + bx + c = d. In algebra, the term with highest exponent is written in the starting and further the terms are written with reducing powers. In the above image ax2 + bx + c = d, there are 4 terms. An algebraic equation may have different terms which are like or unlike. Like terms in an equation are the ones which constitute same variables and exponents. On the other hand, unlike terms in an equation constitute different variables and exponents. Algebra Rules There are five basic rules of algebra. They are: Commutative Rule of Addition Commutative Rule of Multiplication Associative Rule of Addition Associative Rule of Multiplication Distributive Rule of Multiplication Commutative Rule of Addition In algebra, the commutative rule of addition states that when two terms are added, the order of addition does not matter. The equation for the same is written as, (a + b) = (b + a). For example, (x3 + 2x) = (2x + x3) Commutative Rule of Multiplication The commutative rule of multiplication states that when two terms are multiplied, the order of multiplication does not matter. The equation for the same is written as, (a × b) = (b × a). For example, (x4 - 2x) × 3x = 3x × (x4 - 2x). LHS = (x4 - 2x) × 3x = (3x5 - 6x2) RHS = 3x × (x4 - 2x) = (3x5 - 6x2) Here, LHS = RHS, this proves that their values are equal. Associative Rule of Addition In algebra, the associative rule of addition states that when three or more terms are added, the order of addition does not matter. The equation for the same is written as, a + (b + c) = (a + b) + c. For example, x5 + (3x2 + 2) = (x5 + 3x2) + 2 Associative Rule of Multiplication Similarly, the associative rule of multiplication states that when three or more terms are multiplied, the order of multiplication does not matter. The equation for the same is written as, a × (b × c) = (a × b) × c. For example, x3 × (2x4 × x) = (x3 × 2x4) × x. Distributive Rule of Multiplication The distributive rule of multiplication states that when we multiply a number to addition of two numbers, it results in the output which is same as the sum of their products with the number individually. This is distribution of multiplication over addition. The equation for the same is written as, a × (b + c) = (a × b) + (a × c). For example, x2 × (2x + 1) = (x2 × 2x) + (x2× 1). Algebraic Operations The four basic algebraic operations are: Addition Subtraction Multiplication Division In each of the algebraic operations performed, we always categorize the terms in our algebraic equations as like and unlike terms. Addition When two or more terms in an algebraic equation are separated by a plus sign "+", the algebraic operation is addition. We always add the like terms and unlike terms separately as they are treated as two different quantities. Mathematically two different quantities cannot be added together. Example of like terms addition: 5b + 3b = 8b Example of unlike terms addition: 25x + 35y As we can see in the examples, the like terms when added give the same term while the unlike terms cannot be added any further. Subtraction When two or more terms in any algebraic equation are separated by a minus sign "-", the algebraic operation is subtraction. Just as in case of addition, the terms are differentiated as like or unlike terms and then subtracted further. Example of like terms subtraction: 3x2 - x2 = 2x2 Example of unlike terms subtraction: 6bc - 9ab Multiplication When two or more terms in an algebraic equation are separated by a multiplication sign "×", the algebraic operation performed is multiplication. While multiplying the like terms or unlike terms we use Laws of Exponents. Example of like terms multiplication: 16f × 4f = 64f2 Example of unlike terms multiplication: x × y3 = xy3 Division When two or more terms in any algebraic equation are separated by a division sign "/", the algebraic operation performed is division. While dividing the like terms, the similar terms can be simplified while for the case of unlike terms, the terms cannot be simplified any further easily. Example of like terms division: 8b/2b = 4 Examples of unlike terms division: x2/2y2 Algebraic Formulas The algebraic formulas that are used more often and must be kept in knowledge are: Topics Related to Basics of Algebra FAQs on Basics of Algebra The basic rules in algebra are: Commutative Rule of Addition Commutative Rule of Multiplication Associative Rule of Addition Associative Rule of Multiplication Distributive Rule of Multiplication What is the Golden Rule in Algebra? The golden rule in algebra is to keep both sides of the equation balanced, i.e; whatever operation is being used on one side of equation, the same will be used on the other side too. What are the Four Algebraic Operations? Addition Subtraction Multiplication Division How Do You Add and Subtract Like Terms? When like terms are added or subtracted, the coefficients are added or subtracted and written before the like terms. Can We Add or Subtract Two Unlike Terms? No, we cannot add or subtract two unlike terms. algebrarules.com Abridged Glossary of Terms absolute value The positive value for a real number, disregarding the sign. Written |x|. For example, |3|=3, |-4|=4, and |0|=0. addition The process of adding two numbers to obtain their sum. arithmetic The type of mathematics that studies how to solve problems involving numbers (but no variables). base In the expression xy, x is called the base and y is the exponent. common denominator A multiple shared by the denominators of two or more fractions. denominator In the fraction x/y, x is called the numerator and y is called the denominator. distributive law The formula a(x+y)=ax+ay. dividend In the expression "a divided by b", a is the dividend and b is the divisor. division A basic arithmetical operation determining how many times one quantity is contained within another. divisor In the expression "a divided by b", a is the dividend and b is the divisor. divisor The nonzero integer d is a divisor of the integer n if n/d is an integer. equation A statement that two expressions are equal to each other. even number An integer that is divisible by 2. exponent In the expression xy, x is called the base and y is called the exponent. factor (noun) An exact divisor of a number. Example: 7 is a factor of 28, because 28 can be divided by 7. factor (verb) To find the factors of a number. factorial n! (read n factorial) is equal to the product of the integers from 1 to n. fraction An expression of the form a/b. greatest common divisor The greatest common divisor of a sequence of integers, is the largest integer that divides each of them exactly. greatest common factor Same as greatest common divisor. hypotenuse The longest side of a right triangle. imaginary number A complex number of the form xi where x is real and i=sqrt(-1). imaginary part The imaginary part of a complex number x+iy where x and y are real is y. inequality The statement that one quantity is less than (or greater than) another. irrational number A number that is not rational. least common multiple The least common multiple of a set of integers is the smallest integer that is an exact multiple of every number in the set. lowest common denominator The smallest number that is exactly divisible by each denominator of a set of fractions. lowest terms A fraction is said to be in lowest terms if its numerator and denominator have no common factor. multiplication The basic arithmetical operation of repeated addition. numerator In the fraction x/y, x is called the numerator and y is called the denominator. odd number An integer that is not divisible by 2. percent A way of expressing a number as a fraction of 100. perfect cube An integer is a perfect cube if it is of the form m3 where m is an integer. perfect power An integer is a perfect power if it is of the form mn where m and n are integers and n>1. perfect square An integer is a perfect square if it is of the form m2 where m is an integer. power A number multiplied by itself a specified number of times. prime A prime number is an integer larger than 1 whose only positive divisors are 1 and itself. product The result of multiplying two numbers. quotient The result of a division. rational number A rational number is a number that is the ratio of two integers. All other real numbers are said to be irrational. real part The real number x is called the real part of the complex number x+iy where x and y are real and i=sqrt(-1). remainder The number left over when one number is divided by another. repeating decimal A decimal whose digits eventually repeat. right angle An angle formed by two perpendicular lines; a 90 degree angle. right triangle A triangle that contains a right angle. subtraction A basic operation of arithmetic in which you take away one number from another. sum The result of adding two or more numbers. « Back to The Rules of Algebra , the free encyclopedia that anyone can edit. 110,331 active editors 7,014,512 articles in English Igor Judge, Baron Judge, a British judge Nominative determinism is the hypothesis that people are drawn to professions that fit their name. The term was first used in the magazine New Scientist in 1994, after its humorous Feedback column mentioned a book on polar explorations by Daniel Snowman and an article on urology by researchers named Splatt and Weedon. The hypothesis had been suggested by psychologist Carl Jung, citing as an example Sigmund Freud (German for 'joy'), who studied pleasure. A few recent empirical studies have indicated that certain professions are disproportionately represented by people with appropriate surnames, though the methods of these studies have been challenged. One explanation for nominative determinism is the theory of implicit egotism, which states that humans have an unconscious preference for things they associate with themselves. An alternative explanation is genetic: an ancestor might have been named Smith or Taylor according to their occupation, and the genes they passed down might correlate to aptitudes for those professions. (Full article...) Recently featured: Donkey Kong Land History of education in Wales (1701–1870) White dwarf Archive By email More featured articles About Congregation Shaar Hashomayim ... that the Congregation Shaar Hashomayim (pictured) in Windsor, Ontario, was modelled after a synagogue in Montreal? ... that NASA promoted the "faster, better, cheaper" approach to spacecraft missions in the 1990s? ... that Black model Debra Shaw walked the runway in Bellmer La Poupée wearing a metal frame that generated controversy over a perceived reference to slavery? ... that a spokesman for the Kaw Nation credited Robert L. Rankin with single-handedly preserving their language? ... that multiple members of various royal families have competed in the Olympics? ... that The Westerlies folded a piece of tin foil over the bell of a trombone to make it hiss? ... that after runner Frej Liewendahl had broken Paavo Nurmi's five-year winning streak, he went to his hotel room with flowers to apologise? ... that the crew of the US Coast Guard Cutter Dione repeatedly attacked the shipwrecks of oil tankers, believing them to be German U-boats? ... that one issue facing tsunami sirens in New Zealand has been a youth subculture that steals sirens to compete for the loudest and clearest sound? Archive Start a new article Nominate an article Trifid and Lagoon nebulae The Vera C. Rubin Observatory in Chile releases the first light images (example shown) from its new 8.4-metre (28 ft) telescope. In basketball, the Oklahoma City Thunder defeat the Indiana Pacers to win the NBA Finals. An attack on a Greek Orthodox church in Damascus, Syria, kills at least 25 people. The United States conducts military strikes on three nuclear facilities in Iran. In rugby union, the Crusaders defeat the Chiefs to win the Super Rugby Pacific final. Ongoing: Gaza war Iran–Israel war Russian invasion of Ukraine timeline Sudanese civil war timeline Recent deaths: John R. Casani Richard Gerald Jordan Franco Testa Raymond Laflamme Gertrud Leutenegger Maria Voce Nominate an article June 27: Helen Keller Day in the United States Depiction of Pope Agatho 678 – Pope Agatho (depicted), later venerated as a saint in both the Catholic and Eastern Orthodox churches, began his pontificate. 1800 – War of the Second Coalition: French forces won a victory at the Battle of Neuburg, ending Austrian control over the River Danube. 1905 – First Russian Revolution: The crew of the Russian battleship Potemkin began a mutiny against their officers. 1950 – Korean War: Five North Korean aircraft attacked an American air convoy above Suwon Air Base in the first air engagement of the Korean War. 2015 – Ignition of corn starch caused a dust fire at a water park in New Taipei City, Taiwan, killing 15 people and injuring more than 400 others. Wilhelmina FitzClarence, Countess of Munster (b. 1830)Frank Rattray Lillie (b. 1870)Harry Pollitt (d. 1960)Nico Rosberg (b. 1985) More anniversaries: June 26 June 27 June 28 Archive By email List of days of the year About 2024 variant of the Men's T20 World Cup logo The ICC Men's T20 World Cup, formerly the ICC World Twenty20, is a biennial world cup for cricket in the Twenty20 International (T20I) format, organised by the International Cricket Council (ICC). It was held in every odd year from 2007 to 2009, and since 2010 has been held in every even year with the exception of 2018 and 2020. In 2018, the tournament was rebranded from the World Twenty20 to the Men's T20 World Cup. As of the 2024 tournament, twenty-four nations have played in the T20 World Cup. Nine teams have competed in every tournament, six of which have won the title. The West Indies, England and India have won the title twice each, while Pakistan, Sri Lanka and Australia have won the title once each. Sri Lanka, England, Pakistan and India have each made three final appearances, while Pakistan have also made six semi-final appearances. The best result by a non-Test playing nation is the second round appearance by the United States in 2024, while the worst result by a Test playing nation is the second round appearance by Zimbabwe in 2022. (Full list...) Recently featured: Michelin-starred restaurants in Turkey Timeline of Partygate Songs by Gen Hoshino Archive More featured lists Whitehead's trogon (Harpactes whiteheadi) is a species of bird in the family Trogonidae. It is endemic to the island of Borneo, where it is an uncommon resident in primary mountain forest. One of Borneo's largest trogons, at 29 to 33 centimetres (11 to 13 inches) long, it is sexually dimorphic. The male is crimson on the head, nape, and underparts, with a black throat and grey chest; the rest of its upperparts are cinnamon-coloured. The female is similarly patterned, but cinnamon-brown where the male is scarlet. The species is primarily an insectivore, but also eats various plant materials, including fruits and seeds. Other than the timing of its breeding, typically between April and June, little is known about its breeding biology. It is classified as a near-threatened species, with population numbers thought to be declining and habitat loss a key threat. This male Whitehead's trogon was photographed perching on a branch near Mount Kinabalu in the Malaysian state of Sabah. Photograph credit: John Harrison Recently featured: Atacamite Turban Head eagle Springbok Archive More featured pictures Community portal – The central hub for editors, with resources, links, tasks, and announcements. Village pump – Forum for discussions about Wikipedia itself, including policies and technical issues. Site news – Sources of news about Wikipedia and the broader Wikimedia movement. Teahouse – Ask basic questions about using or editing Wikipedia. Help desk – Ask questions about using or editing Wikipedia. Reference desk – Ask research questions about encyclopedic topics. Content portals – A unique way to navigate the encyclopedia. Wikipedia is written by volunteer editors and hosted by the Wikimedia Foundation, a non-profit organization that also hosts a range of other volunteer projects: CommonsFree media repository MediaWikiWiki software development Meta-WikiWikimedia project coordination WikibooksFree textbooks and manuals WikidataFree knowledge base WikinewsFree-content news WikiquoteCollection of quotations WikisourceFree-content library WikispeciesDirectory of species WikiversityFree learning tools WikivoyageFree travel guide WiktionaryDictionary and thesaurus This Wikipedia is written in English. Many other Wikipedias are available; some of the largest are listed below. 1,000,000+ articles العربية Deutsch Español فارسی Français Italiano Nederlands 日本語 Polski Português Русский Svenska Українська Tiếng Việt 中文 250,000+ articles Bahasa Indonesia Bahasa Melayu Bân-lâm-gú Български Català Čeština Dansk Eesti Ελληνικά Esperanto Euskara עברית Հայերեն Magyar Norsk bokmål Română Simple English Slovenčina Srpski Srpskohrvatski Suomi Türkçe Oʻzbekcha 50,000+ articles Asturianu Azərbaycanca [illegible] Bosanski کوردی Frysk Gaeilge Galego Hrvatski ქართული Kurdî Latviešu Lietuvių [illegible] Македонски [illegible] Norsk nynorsk [illegible] Shqip Slovenščina [illegible] اردو Retrieved from " 2 This article is about the year 678. For the film, see 678 (film). For the Argentine TV program, see 6, 7, 8. Calendar year Years Millennium 1st millennium Centuries 6th century 7th century 8th century Decades 650s 660s 670s 680s 690s Years 675 676 677 678 679 680 681 vte 678 by topic Leaders Political entities State leaders Religious leaders Categories Births Deaths Establishments vte 678 in various calendarsGregorian calendar678DCLXXVIIIAb urbe condita1431Armenian calendar127ԹՎ ՃԻԷAssyrian calendar5428Balinese saka calendar599–600Bengali calendar84–85Berber calendar1628Buddhist calendar1222Burmese calendar40Byzantine calendar6186–6187Chinese calendar丁丑年 (Fire Ox)3375 or 3168 — to —戊寅年 (Earth Tiger)3376 or 3169Coptic calendar394–395Discordian calendar1844Ethiopian calendar670–671Hebrew calendar4438–4439Hindu calendars - Vikram Samvat734–735 - Shaka Samvat599–600 - Kali Yuga3778–3779Holocene calendar10678Iranian calendar56–57Islamic calendar58–59Japanese calendarHakuchi 29(白雉２９年)Javanese calendar570–571Julian calendar678DCLXXVIIIKorean calendar3011Minguo calendar1234 before ROC民前1234年Nanakshahi calendar−790Seleucid era989/990 AGThai solar calendar1220–1221Tibetan calendar阴火牛年(female Fire-Ox)804 or 423 or −349 — to —阳土虎年(male Earth-Tiger)805 or 424 or −348 Pope Agatho I (678–681) Year 678 (DCLXXVIII) was a common year starting on Friday of the Julian calendar. The denomination 678 for this year has been used since the early medieval period, when the Anno Domini calendar era became the prevalent method in Europe for naming years. July 27 – The Siege of Thessalonica (676–678) ends, when the Sclaveni withdraw. Autumn – Siege of Constantinople: Emperor Constantine IV confronts the Arab besiegers in a head-on engagement. The Byzantine fleet, equipped with Greek fire, destroys the Muslim fleet at Sillyon,[1][2][3][4] ending the Arab threat to Europe, and forcing Yazid (a son of caliph Muawiyah I) to lift the siege on land and sea. The victory also frees up forces that are sent to raise the two-year siege of Thessalonica by the local Slavic tribes. King Æthelred of Mercia defeats the Northumbrian forces under King Ecgfrith, in a battle near the River Trent. Archbishop Theodore helps to resolve differences between the two, Æthelred agreeing to pay a weregild to avoid any resumption of hostilities (approximate date). April 27 – Emperor Tenmu holds divination for the purpose of proceeding to the Abstinence Palace. May 3 – Princess Tōchi suddenly takes ill and dies within the palace. Tenmu, her father, is unable to sacrifice to the Gods of Heaven and Earth. May 10 – Tōchi is buried at a place which could be Akō (Hyōgo Prefecture). Tenmu is graciously pleased to raise lament for her. Wilfrid, bishop of York, is at the height of his power and owns vast estates throughout Northumbria. After his refusal to agree to a division of his see, Ecgfrith and Theodore, archbishop of Canterbury, have him banished from Northumbria. April 11 – Pope Donus dies at Rome, after a reign of 1 year and 160 days. He is succeeded by Agatho I, who becomes the 79th pope. He is the first pope to stop paying tribute to Emperor Constantine IV upon election. In Japan, the national worshiping to the Gods of Heaven and Earth is planned. Tenmu tries to select his daughter Tōchi as a Saiō to make her serve the Gods. However, Tōchi suddenly takes ill and dies. The Beomeosa temple complex in Geumjeong-gu (modern South Korea) is constructed, during the reign of King Munmu of Silla. Childebert III, Merovingian Frankish king and son of Theuderic III Childebrand I, duke of Burgundy (d. 751) K'inich Ahkal Mo' Nahb III, Maya ruler of Palenque April 11 – Pope Donus May 3 – Tōchi, Japanese princess Abdullah ibn Aamir, Arab general (b. 626) Ælfwine, king of Deira (approximate date) Aisha, wife of Muhammad Arbogast, bishop of Strasbourg Nathalan, Scottish bishop Wechtar, Lombard duke of Friuli Zhang Wenguan, chancellor of the Tang dynasty (b. 606) ^ Haldon 1990, p. 64. ^ Lilie 1976, pp. 78–79. ^ Treadgold 1997, pp. 326–327. ^ Mango & Scott 1997, p. 494. Haldon, John F. (1990). Byzantium in the Seventh Century: The Transformation of a Culture (revised ed.). Cambridge University Press. ISBN 978-0-521-31917-1. Lilie, Ralph-Johannes (1976). Die byzantinische Reaktion auf die Ausbreitung der Araber. Studien zur Strukturwandlung des byzantinischen Staates im 7. und 8. Jhd [Byzantine Reaction to the Expansion of the Arabs. Studies on the Structural Change of the Byzantine State in the 7th and 8th Cent.] (in German). Munich: Institut für Byzantinistik und Neugriechische Philologie der Universität München. OCLC 797598069. Mango, Cyril; Scott, Roger (1997). The Chronicle of Theophanes Confessor. Byzantine and Near Eastern History, AD 284–813. Oxford University Press. ISBN 0-19-822568-7. Treadgold, Warren (1997). A History of the Byzantine State and Society. Stanford, California: Stanford University Press. ISBN 0-8047-2630-2. Retrieved from " 32009 Argentine TV series or program 6, 7, 8First logoAlso known asThe Critique to Real PowerGenreArchive television programPresented by Orlando Barone Carlos Barragán Jorge Dorio Cynthia García Eduardo Massa Alcántara Mariana Moyano Dante Palma Sandra Russo Nora Veiras Country of originArgentinaOriginal languageSpanishNo. of seasons7ProductionExecutive producerDiego GvirtzProducerPensado Para TelevisiónOriginal releaseNetworkTelevisión Pública ArgentinaReleaseMarch 9, 2009 (2009-03-09) –23 December 2015 (2015-12-23) Six in the Seven at Eight, usually called 6, 7, 8, was an Argentine political commentary TV program broadcast by the government-run Channel 7 since 2009. Its name comes from the fact that, when it first started airing, there were five members on the show's panel, and its motto was you are the sixth one (the viewer). Since it was broadcast by Channel 7 at 8 p.m., the name was shortened to "6, 7, 8". In late 2009, the program was moved to 9 p.m., a new segment was added to be aired on Sundays at night, and new guest panelists were invited, making it more than six members.[1] Nevertheless, the show's name remained unchanged. It was first hosted by María Julia Oliván and a panel which included Orlando Barone, Carla Czudnowsky, Eduardo Cabito Massa Alcántara, Luciano Galende and Sandra Russo, along with a guest analyst who would give their opinions throughout the program. María Julia Oliván announced in an interview for web site Television.com.ar that, on January 28, 2010, should make her last appearance on the program.[2] Her place was occupied by Luciano Galende, and from 2013 onward the host has been Carlos Barragán. The program was nominated to the 2010 Martín Fierro Awards in the category Best Journalistic Program. The program's panel discussed national and world events and analyzed their social and political repercussions on Argentina. The program has come under criticism due to its perceived advocacy of Kirchnerism, which is controversial since it is aired by a state-owned TV channel during the time when Cristina Kirchner was president. The program ceased to be aired shortly after the end of the presidency of Cristina Kirchner. 6, 7, 8 was first aired on March 9, 2009. The initial name was "6 in the 7 at 8", making reference to the 6 people in the program, the channel Televisión Pública Argentina that airs in channel 7, and that the program was broadcast at 8:00 pm. Although the number of people and the timeslot changed over time, the name "678" was kept. It is an Archive television program focused on politics and journalism. It was created during the campaign for the 2009 midterm elections, in order to broadcast the Kirchnerite propaganda known as Relato K, and to attack the opposing candidates. It received a privileged timeslot on Sundays, right after the broadcasting of the Fútbol para todos weekly matches. Once the elections were over, it continued to voice the Kirchnerite view in the conflict between Kirchnerism and the media.[3] Mauricio Macri, president of Argentina since December 10, 2015, appointed Hernán Lombardi as the new manager for Channel 7. Soon thereafter it was announced that the channel would not air 6, 7, 8 because the production company decided not to renew the contract with the TV Pública.[4][5] According to Clarín newspaper the program uses archive footage to criticize Mass Media outlets, judges and political opponents to the national government.[6] On October 13, 2009 the program aired a video that had circulated in blogs. The anonymous video was recorded through a hidden camera, and it shows the journalist and columnist of newspaper La Nación, Carlos Pagni, in an alleged operation to publish false information for the purpose of damaging the oil company Repsol YPF. The broadcast of the video was criticized by the Partido Solidario deputy Carlos Heller who was a guest on the program that day, expressing his objection to the publication of anonymous films. The contents of the video were criticized by the panelists after it was shown. According to an article in La Nación that was published the next day about the segment, "the presentation of the hidden camera, and the images of the hidden camera (cut, but carefully and professionally edited) do not, at any moment, show the columnist in situations that could corroborate the serious and injurious charges about corruption that are made in the video through printed boards and a voice-over".[7] In the video, there are appearances by other people who may represent Pagni, receiving money in return for newspaper articles. However, after the airing on October 13, Pagni received the support of the Argentine Journalism Forum (Foro del Periodismo Argentino) and other journalists.[8] YPF issued a complaint to investigate who recorded the video, and the veracity of the facts that are seen on it. They assured that "it is true that the video is anonymous and made in a more obscure way. However, it warns that it was a journalistic operation against us".[9] 2013 Martín Fierro Awards Best journalism program[10] ^ Asteriscos.tv Seis, siete, ocho ¿y ahora a las nueve? (in Spanish) ^ "6, 7, 8 en TVPública.com.ar". Archived from the original on 2013-08-04. Retrieved 2010-04-16. ^ Oliván, pp. 9-12 ^ "Chau 6, 7, 8". Archived from the original on 2015-11-25. Retrieved 2015-11-24. ^ "Adiós 678". ^ Clarin.com, «El programa que ataca a los medios críticos cuesta caro» Archived 2010-01-13 at the Wayback Machine (in Spanish) ^ La Nación.com, «Agravia Canal 7 a un columnista con un video anónimo» Archived 2011-06-05 at the Wayback Machine (in Spanish) ^ Cable en Minutouno Archived 2011-07-06 at the Wayback Machine (in Spanish) ^ "Igual hubo una operación", por Raúl Kollmann (in Spanish) ^ "Todos los nominados a los Martín Fierro 2014" [All the nominations for the 2014 Martín Fierro]. La Nación (in Spanish). April 15, 2014. Retrieved April 15, 2014. Oliván, María Julia (2010). 678: La creación de otra realidad. Argentina: Paidos. ISBN 978-950-12-0700-2. Website for the company Pensado para Televisión Official website of Canal Siete Retrieved from " 4 The following pages link to 6, 7, 8 External tools (link count transclusion count sorted list) · See help page for transcluding these entries Showing 22 items. View (previous 50 | next 50) (20 | 50 | 100 | 250 | 500)678 (links | edit) Index of Argentina-related articles (links | edit) Orlando Barone (links | edit) 678 show (redirect page) (links | edit) Six in the Seven at Eight (redirect page) (links | edit) 44th Martín Fierro Awards (links | edit) 43rd Martín Fierro Awards (links | edit) 45th Martín Fierro Awards (links | edit) Relato K (links | edit) 6-7-8 (redirect page) (links | edit) Talk:6-7-8 (links | edit) User:EranBot/Copyright/rc/62 (links | edit) Archive television program (links | edit) Talk:6, 7, 8 (transclusion) (links | edit) User:AlexNewArtBot/TelevisionSearchResult/archive2 (links | edit) User:AlexNewArtBot/FilmsSearchResult/archive19 (links | edit) User:AlexNewArtBot/LawFeedSearchResult/archive76 (links | edit) User:AlexNewArtBot/TestFeedSearchResult/archive115 (links | edit) User:AlexNewArtBot/ArgentinaSearchResult/archive3 (links | edit) User:CleanupWorklistBot/lists/Argentina (links | edit) User:Uanfala/dab/missing entries exceptions (links | edit) Wikipedia:WikiProject Argentina/Politics watchlist (links | edit) Wikipedia:WikiProject Latin America/The 10,000 Challenge (links | edit) Wikipedia:WikiProject Latin America/The 10,000 Challenge/1–1000 (links | edit) View (previous 50 | next 50) (20 | 50 | 100 | 250 | 500) Retrieved from " WhatLinksHere/6,_7,_8" -(a\pm b)=-a\mp b a(b+c)=ab+ac a(b+c)(d+e)=abd+abe+acd+ace (a+b)(c+d)=ac+ad+bc+bd -(-a)=a \frac{0}{a}=0 \: ,\: ae 0 \frac{a}{1}=a \frac{a}{a}=1 (\frac{a}{b})^{-1}=\frac{1}{\frac{a}{b}}=\frac{b}{a} (\frac{a}{b})^{-c}=((\frac{a}{b})^{-1})^{c}=(\frac{b}{a})^{c} a^{-1}=\frac{1}{a} a^{-b}=\frac{1}{a^b} \frac{-a}{-b}=\frac{a}{b} \frac{-a}{b}=-\frac{a}{b} \frac{a}{-b}=-\frac{a}{b} \frac{a}{\frac{b}{c}}=\frac{a\cdot c}{b} \frac{\frac{b}{c}}{a}=\frac{b}{c \cdot a} \frac{1}{\frac{b}{c}}=\frac{c}{b} \left| -a \right| = \left| a \right| \left|a\right|=a \: ,\: a\ge0 \left| ax\right| = a \left| x\right| \: , \: a\ge 0 1^{a}=1 a^{1}=a a^{0}=1\:,\: ae 0 0^{a}=0\:,\: ae 0 (ab)^n=a^{n}b^{n} \frac{a^m}{a^n}=a^{m-n}\:,\: m>n \frac{a^m}{a^n}=\frac{1}{a^{n-m}}\:,\: n>m a^{b+c}=a^{b}a^{c} (a^{b})^{c}=a^{b\cdot c} a^{bx}=(a^b)^x (\frac{a}{b})^{c}=\frac{a^{c}}{b^{c}} a^c \cdot b^c=(a\cdot b)^{c} \sqrt{1}=1 \sqrt{0}=0 \sqrt[n]{a}=a^{\frac{1}{n}} \sqrt[n]{a^m}=a^{\frac{m}{n}} \sqrt{a}\sqrt{a}=a \sqrt[n]{a^n}=a,\:a\ge0 \sqrt[n]{a^n}=|a|,\:\mathrm{n\:is\:even} \sqrt[n]{a^n}=a,\:\mathrm{n\:is\:odd} \sqrt[n]{ab}=\sqrt[n]{a}\sqrt[n]{b},\:a,b\ge0 \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}},\:a,b\ge0 x^{2}-y^{2} = (x-y)(x+y) x^{3}+y^{3} = (x+y)(x^{2}-xy+ y^{2}) x^{n}-y^{n} = (x-y)(x^{n-1}+x^{n-2}y+ \dots + xy^{n-2} + y^{n-1}) x^{n}+y^{n} = (x+y)(x^{n-1}-x^{n-2}y+ \dots - xy^{n-2} + y^{n-1}) \quad \quad \mathrm{n\:is\:odd} ax^(2n)-b = (\sqrt{a}x^n+\sqrt{b})(\sqrt{a}x^n-\sqrt{b}) ax^(4)-b = (\sqrt{a}x^2+\sqrt{b})(\sqrt{a}x^2-\sqrt{b}) ax^(2n)-by^(2m) = (\sqrt{a}x^n+\sqrt{b}y^m)(\sqrt{a}x^n-\sqrt{b}y^m) ax^(4)-by^(4) = (\sqrt{a}x^2+\sqrt{b}y^2)(\sqrt{a}x^2-\sqrt{b}y^2) \frac{n!}{(n+m)!}=\frac{1}{(n+1)\cdot(n+2)\cdots(n+m)} \frac{n!}{(n-m)!}=n\cdot(n-1)\cdots(n-m+1), n>m 0!=1 n!=1\cdot2\cdots(n-2)\cdot(n-1)\cdot n \log(1)=0 \log_a(a)=1 \log_{a}(x^b)=b\cdot\log_{a}(x) \log_{a^b}(x)=\frac{1}{b}\log_{a}(x) \log_{a}(\frac{1}{x})=-\log_{a}(x) \log_{\frac{1}{a}}(x)=-\log_{a}(x) \log_{a}(b)=\frac{\ln(b)}{\ln(a)} \log_{x}(x^n)=n \log_{x}((\frac{1}{x})^{n})=-n a^{\log_{a}(b)}=b 0^{0}=\mathrm{Undefined} \frac{x}{0}=\mathrm{Undefined} \log_{a}(b)=\mathrm{Undefined}\:,\: a\le0 \log_{a}(b)=\mathrm{Undefined}\:,\: b\le0 \log_{1}(a)=\mathrm{Undefined} i^{2}=-1 \sqrt{-1}=i \sqrt{-a}=\sqrt{-1}\sqrt{a} FREE BOOKS EVERYDAY!Join 100,000+ book lovers and get exclusive free books delivered to your inbox every day! Discover new stories daily and grow your library—completely free!ALL AUTHORS >SEE ALL BOOKS > by InfoBooks - Updated: January 2025Discover the universal language of mathematics with our collection of free PDF algebra books.Algebra is a fundamental branch of mathematics, essential for understanding structures, relationships, and abstract concepts applied in various scientific and technological areas.From basic concepts to advanced theories, our selection is designed for students, educators, and anyone interested in deepening their mathematical knowledge.By providing these books for free and without registration, we facilitate access to quality mathematical education, removing barriers and fostering continuous learning.Download our collection of algebra books in PDF for free and immerse yourself in the study of this fascinating and challenging discipline.Start exploring the world of algebra.Algebra BooksExtension: PDF | 1331 pagesIntermediate Algebra 2e, this book is a comprehensive textbook designed to enhance student understanding of algebraic concepts, covering topics from basic foundations to advanced functions.Extension: PDF | 619 pagesCollege Algebra, this document is a comprehensive textbook on College Algebra designed to enhance student access to quality learning materials. 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These basic rules are useful for everything from figuring out your gas mileage to acing your next math test — or even solving equations from the far reaches of theoretical physics. Happy calculating! ```{\left({a \over c}\right) \over b} = {a \over bc}``` Description:If you divide the numerator by a particular number, it has the same effect on the fraction's overall value as if you multiply the denominator by that same number. A division above equals a multiplication below. ```{ \left({1 \over 5}\right) \over 2} = {1 \over 10} = {1 \over (2*5)}``` « Previous Rule Next Rule » Algebra rules is a project by two of the folks who run The Autodidacts. A couple of autodidact math enthusiasts, we were looking for all the rules of basic algebra concisely presented in one place. We couldn't find such a place, so we made Algebrarules.com These simple rules — applied with a pinch of imagination and a dash of arithmetic — can divide, conquer, and solve just about any practical algebra problem. If you find errata in the math, bugs in the code of Algebrarules.com, or just want to say Eh, please send us a letter or join us on our roost: @rulesofalgebra. You've made it all the way to the end! If you found this site useful, pay it forward by helping us help more people learn algebra! X Howdy! Here are a few very handy rules of algebra. These basic rules are useful for everything from figuring out your gas mileage to acing your next math test — or even solving equations from the far reaches of theoretical physics. Happy calculating! ```(a^n)^m = a^{nm}``` Description:Like the previous rule, this one can be demonstrated simply by expanding the exponents out into a series of multiplications ```(4^2)^3 = (4*4)^3 = (4*4)(4*4)(4*4) = 4*4*4*4*4*4 = 4^6 = 4^{2*3}``` « Previous Rule Next Rule » Algebra rules is a project by two of the folks who run The Autodidacts. A couple of autodidact math enthusiasts, we were looking for all the rules of basic algebra concisely presented in one place. We couldn't find such a place, so we made Algebrarules.com These simple rules — applied with a pinch of imagination and a dash of arithmetic — can divide, conquer, and solve just about any practical algebra problem. If you find errata in the math, bugs in the code of Algebrarules.com, or just want to say Eh, please send us a letter or join us on our roost: @rulesofalgebra. You've made it all the way to the end! If you found this site useful, pay it forward by helping us help more people learn algebra! Algebra is a branch of Mathematics that substitutes letters for numbers. In the school education, Algebra Formulas form the foundation of numerous topics of mathematics. Topics like equations, quadratic equations, polynomials, coordinate geometry, calculus, trigonometry, and probability extensively depend on algebraic formulas for understanding and for solving complex problems. Download the Algebra Formulas PDF along with all Algebraic Identities Formula Charts in one place from here. Algebra Formula One of the most crucial areas of mathematics is algebra. Numerous disciplines, including quadratic equations, polynomials, coordinate geometry, calculus, trigonometry, probability, and others, can be solved using algebraic formulas. In these formulas, we used numbers along with letters. The most common letters used in algebraic equations and problems are X, Y, A, and B. These formulas enable us to quickly and efficiently tackle time-consuming algebraic problems. Here, we include all significant Algebraic formulas together with their solutions, so that students can access them all in one place. Algebra Formulas Example Algebra formulas are basically algebra equations formed by algebraic and mathematical phrases and symbols. These algebraic formulas contain an unknown variable x, which can be generated while simplifying an equation. These algebraic equations efficiently solve complicated algebraic computations. For example, $(a+b)^3 = a^3 + 3a^2b+3ab^2+b^3$ In the above formulas, both sides are individual algebraic equations. Where ($a^3 + 3a^2b+3ab^2+b^3$) is the simplified expression of $(a+b)^3$. Algebra Formulas Identities In algebra formulas, an identity is an equation that is always true regardless of the values assigned to the variables. Algebraic Identity means that the left-hand side (LHS) of the equation is identical to the right-hand side (RHS) of the equation and for all values of the variables. Algebraic identities are solved for the values of unknown variables. Here are some commonly used algebraic identities: Algebraic Identities Formula (a + b)2 = a2 + 2ab + b2 (a – b)2 = a2 – 2ab + b2 (a + b)(a – b) = a2 – b2 (x + a)(x + b) = x2 + x(a + b) + ab Algebra Formulas For Squares of Class 10 Here are some important formulas involving squares. • $a^2– b^2 = (a – b)(a + b)$ • $(a + b)^2 = a^2 + 2ab + b^2$ • $a^2 + b^2 = (a + b)^2 – 2ab$ • $(a – b)^2 = a^2 – 2ab + b^2$ • $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ • $(a – b – c)^2 = a^2 + b^2 + c^2 – 2ab + 2bc – 2ca$ Algebra Formulas for Cube Here are some Algebraic formulas involving cubes. • $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ • $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ • $(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$ • $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$ • $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$ • $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$ Some more Algebra formulas are – • $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^2$ • $(a – b)^4 = a4 – 4a^3b + 6a^2b^2 – 4ab^3 + b^4$ • $a^4 – b^4 = (a – b)(a + b)(a^2 + b^2)$ • $a^5 – b^5 = (a – b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ Algebra Formula for Natural Numbers Algebra formulas for Natural Numbers. Except for 0 and negative numbers, the rest of the numbers [2 to infinity] in the number system that humans can count are known as Natural Numbers. Some algebraic formulas are applied when performing operations on natural numbers. They are. Consider n to be a natural number. (an – bn)= (a – b)(an-1 + an-2b+…+ bn-2a + bn-1) (an + bn)= (a + b)(an-1 – an-2b +…+ bn-2a – bn-1) [where n is even , (n = k + 1)] (an + bn)= (a + b)(an-1 – an-2b +an-3b2…- bn-2a + bn-1) [where n is odd , (n = 2k + 1)] Laws of Exponents In Algebra, An Exponent or power is used to demonstrate the repeated multiplication of a number. Such as 3×3×3×3 can be written as 3^4 where 4 is the exponent of 3.   In general, exponents or powers indicate how many times a number can be multiplied. There are various rules to operate an exponent for addition, subtraction, and multiplication, which are easily solved by algebraic formulas. Algebra formula: Laws of Exponents Algebra Formulas for Quadric equations Quadratic equations are one of the most important topics in the syllabus of classes 9 and 10. To find the root of the given quadratic equations, we used the following formulas If $ax^2+bx+c =0$ is a quadratic equation, then Algrabra formula:Quadric equations From the above formula we can conclude that , If the roots of the quadric equation are α and β 1. The equation will be (x − α)(x − β) = 0, 2. The value of (α + β) = (-b / a) and α × β = (c / a). Algebra Formulas For Irrational Numbers The formulas used to solve equations based on Irrational Numbers are as follows √ab = √a √b √a/b =√a / √b (√a + √b) (√a – √b) = a-b (√a +√b)² = a + 2 √ab + b (a +√b)(a –√b) = a² – b Algebra Formulas List/Chart Here is a list of all important Algebraic formulas. Students must go through the list to solve difficult algebraic equations very quickly. Important Formulas 1 $a^2– b^2 = (a – b)(a + b)$ 2 $(a + b)^2 = a^2 + 2ab + b^2$ 3 $a^2 + b^2 = (a + b)^2 – 2ab$ 4 $(a – b)^2 = a^2 – 2ab + b^2$ 5 $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ 6 $(a – b – c)^2 = a^2 + b^2 + c^2 – 2ab + 2bc – 2ca$ 7 $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ 8 $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ 9 $(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$ 10 $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$ 11 $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$ 12 $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$ 13 $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^2$ 14 $(a – b)^4 = a4 – 4a^3b + 6a^2b^2 – 4ab^3 + b^4$ 15 $a^4 – b^4 = (a – b)(a + b)(a^2 + b^2)$ 16 $a^5 – b^5 = (a – b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ Some commonly used Algebraic formulas in maths Below, we have shared some derivations of frequently used Algebra formulas for your better understanding. a+b+c Whole Square Formula- (a+b+c)^2 Formula The square of the sum of three terms can be expanded using the formula for the square of a trinomial: (a+b+c)2=a2+b2+c2+2ab+2ac+2bc So, (a+b+c)2 is equal to the sum of the squares of the individual terms (a2, b2, c2) and twice the products of the pairs of terms (2ab, 2ac, 2bc). This expansion holds true for any values ofa, b, andc. a-b Whole Square The square of the difference between two terms can be expanded using the formula for the square of a binomial: (a−b)2=a2−2ab+b2 So, (a−b)2 is equal to the square of the first term (a2), minus twice the product of the two terms (2ab), and the square of the second term (b2). This expansion holds true for any values of andb. a+b Whole Square The square of the sum of two terms, a+b, can be expanded using the formula for the square of a binomial: (a+b)2=a2+2ab+b2 So, (a+b)2 is equal to the sum of the squares of the individual terms (a2 and b2) and twice the product of the terms (2ab). This expansion holds true for any values of a and b. Implementation of Algebra All Formulas with Examples Example 1 : Find the value $20^2- 15^2$ Solution: To solve the equation ,the formula we use is $a^2 -b^2 = (a+b) (a-b)$ = (20+15)(20-15) = 35 × 5 = 175 (Answer) Example 2 : (x-y) =2 and $x^2+ y^2$ =20 then find the value of x and y [where x,y >0] Solution: Here , $x^2+ y^2 =20$ $(x-y)^2 +2xy =20$ Or, $(2)^2+ 2xy =20$ Or, 2xy = 20-4 = 16 Or,xy = 8 Now, $(x+y)^2= (x-y)^2 +4xy = (2)^2 +4.8 =36$ Or,x+y = ± 6 So, x+y = 6 [x,y >0] ....(1) We also get , x- y =2 ......(2) Solving two equations we get , x = 4 and y = 2 (Answer) Example 3 : Divide($a^3 + b^3 + c^3$ – 3abc)by(a+b+c)and the quotient.Determine the magnitude. Solution: $a+b^3$ + c3 – 3abc =$(a+b+c)(a^2+b^2+c^2$-ab-ac-bc) Determinant quotient = $[(a+b+c)(a^2+b^2+c^2$-ab-ac-bc)] ÷ (a+b+c) = $a^2+b^2+c^2$-ab-be-ca. Magnitude of quotient is 2. (Answer) Example 4 Find their successive product (x + y), (x – y), $(x^2 + y^2)$. Solution :Determinant serial product = (x + y) (x – y) $(x^2 + y^2) = (x^2-y^2)(x^2 + y^2) = (x^2)^2 – (y^2)^2 = x^4 – y^4$.(Answer) Some Questions of Algebra Formula 1. If x+y = 3 and xy = 2, what is the value of $(x – y)^2$? 2. If a+b = 8 and ab = 15, what will be the values of a and b? 3. If a+b = 5 and ab = 6, Find the value of $a^2 – b^2$? 4. If x = 29 and y = 14, what is the value of $(4x^2 + 9y^2 + 12xy)$? All Algebraic Identities Algebraic identities are equations that are true for all values of the variables involved. Here are some of the most important algebraic identities: 1. Basic Algebraic Identities Square of a Sum: (a+b)2=a2+2ab+b2(a + b)^2 = a^2 + 2ab + b^2   Square of a Difference: (a−b)2=a2−2ab+b2(a - b)^2 = a^2 - 2ab + b^2   Product of a Sum and Difference: (a+b)(a−b)=a2−b2(a + b)(a - b) = a^2 - b^2   Cubic of a Sum: (a+b)3=a3+3a2b+3ab2+b3(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3   Cubic of a Difference: (a−b)3=a3−3a2b+3ab2−b3(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3   Sum of Cubes: a3+b3=(a+b)(a2−ab+b2)a^3 + b^3 = (a + b)(a^2 - ab + b^2)   Difference of Cubes: a3−b3=(a−b)(a2+ab+b2)a^3 - b^3 = (a - b)(a^2 + ab + b^2)   2. Multinomial Algebraic Identities Square of a Trinomial: (a+b+c)2=a2+b2+c2+2ab+2bc+2ca(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca   Cubic of a Trinomial: (a+b+c)3=a3+b3+c3+3(a+b)(b+c)(c+a)(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)   3. Special Algebraic Identities General Binomial Theorem: (a+b)n=∑k=0n(nk)an−kbk(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k   (a + b + c)² Expanded: (a+b+c)2=a2+b2+c2+2ab+2bc+2ca(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca   Sum of Powers of Roots: an−bn=(a−b)(an−1+an−2b+⋯+bn−1)a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \cdots + b^{n-1})   4. Quadratic Algebraic Identities For any quadratic polynomial: ax2+bx+c=a(x−α)(x−β)ax^2 + bx + c = a(x - \alpha)(x - \beta) where α\alpha and b are the roots of the equation ax2+bx+c=0ax^2 + bx + c = 0 5. Important Algebraic Products Sum of squares of two numbers: a2+b2=(a+b)2+(a−b)22a^2 + b^2 = \frac{(a + b)^2 + (a - b)^2}{2}   Product of four numbers: (a2+b2)(c2+d2)=(ac+bd)2+(ad−bc)2(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2   6. Miscellaneous Algebraic Identities Lagrange's Identity: (a2+b2)(c2+d2)=(ac+bd)2+(ad−bc)2(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2   Expression for (a+b+c)3=a3+b3+c3+3(a+b)(b+c)(c+a)(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a) These identities are useful in simplifying expressions, solving equations, and performing algebraic manipulations.Algebraic Formulas PDF Check Out The Algebraic Formulas PDF for Class 10 Students. Click Here- Math Algebra Formulas PDF Related Post: Learn Algebra almost by accident by putting this meticulously-designed set of educational posters up on your wall! The posters have exact same rules that have helped over 300,000 people since algebrarules.com was launched in 2013. The printed posters let you take them all in at a glance, have them at hand whatever the state of the internet and society, and impress visiting friends. Designed & printed in Canada on FSC-certified recycled paper. PDF Download (coming soon) The poster is almost done! Leave your email and we'll tell you as soon as the PDF is ready. Join the waitlist for the first offset print run of the 3-poster set! X Howdy! Here are a few very handy rules of algebra. These basic rules are useful for everything from figuring out your gas mileage to acing your next math test — or even solving equations from the far reaches of theoretical physics. Happy calculating! a(b+c)=ab+aca(b+c) = ab+ac a(b+c)=ab+ac If you're multiplying something with a sum of two or more other values, you can distribute the multiplication to each of the values, then sum the result. ```3(4+5) = 3*4 + 3*5 = 27``` a(bc)=abca \left({b \over c} \right) = { ab \over c }a(cb)=cab A multiplication or division of the numerator of a fraction affect the fraction as a whole (and vice versa). If you need to multiply a fraction, multiplying the numerator does the job. ```6\left({1 \over 3}\right) = {(6*1) \over 3} = 2``` (ac)b=abc{\left({a \over c}\right) \over b} = {a \over bc}b(ca)=bca If you divide the numerator by a particular number, it has the same effect on the fraction's overall value as if you multiply the denominator by that same number. A division above equals a multiplication below. ```{ \left({1 \over 5}\right) \over 2} = {1 \over 10} = {1 \over (2*5)}``` ``` {a \over \left({b \over c}\right)} = {ac \over b} ``` In the same fashion as the above rule, dividing the denominator of a fraction has the same effect as multiplying the numerator. A fraction below equals a multiplication above. ```{1 \over \left({6 \over 2}\right)} = {(1*2) \over 6} = {1 \over 3}``` ``` \left({a \over b}\right)+\left({c \over d}\right) = {(ad+bc) \over bd} ``` If top and bottom of a fraction are both multiplied by the same number, the fraction remains unchanged. So, we can sum two fractions by first multiplying each fraction's numerator and denominator with the other fraction's denominator. Then, since both the fractions now have the same denominator (the product of the two denominators), we can combine them into one fraction, with the sum in the numerator. ```{3 \over 5}+{1 \over 3} = {(3*3) \over (5*3)} + {(1*5) \over (3*5)} = {(1*5)+(3*3) \over (3*5)} = {14 \over 15}``` ``` \left({a \over b}\right)-\left({c \over d}\right) = {(ad-bc) \over bd} ``` This is another version of rule 5, but for subtraction of two fractions, rather than addition. ```{3 \over 5}-{1 \over 3} = {(3*3) \over (5*3)} - {(1*5) \over (3*5)} = {(3*3)-(1*5) \over (3*5)} = {9-5 \over 15} = {4 \over 15}``` ``` {(a-b) \over (c-d)} = {(b-a) \over (d-c)} ``` Reversing a subtraction gives the inverse result: ``5-3 = 2; 3-5 = -2``. In a fraction, if both the numerator and the denominator are inverted, the value of the fraction stays the same. So, if we reverse a subtraction in both the numerator and denominator, the value of the fraction is unchanged. ```{3-5 \over 2-1} = {-2 \over 1} = {5-3 \over 1-2} = {2 \over -1} = -2``` or ```{-3 \over -4} = 0.75 = {3 \over 4}``` ``` {(a+b) \over c} = {a \over c} + {b \over c} ``` Two fractions with common denominators can be added by adding the numerators and leaving the denominator unchanged. Going the other direction, we can also break apart a fraction with an addition in the numerator into two fractions (each with the common denominator). ```{(1+2) \over 4} = {3 \over 4} = {1 \over 4} + {2 \over 4}``` ``` {ac+bc \over c} = a+b ``` Division is the inverse of multiplication: if ``{a \over b} = c`` then ``b*c = a``. This means that the fraction ``{ac \over c}`` is equal to ``a``, since we are multiplying ``a`` by ``c`` and then immediately dividing it by ``c`` again, which puts us right back where we started. Since we know that ``{ac+bc \over c} = {ac \over c} + {bc \over c}`` (see rule 8), and based on the above we can see that ``{ac \over c} = a`` and ``{bc \over c} = b``, we have our result: ``a+b``. ```{(4*5)+(2*5) \over 5} = {4*5 \over 5}+{2*5 \over 5} = 4+2``` ``` {\left({a \over c}\right) \over \left({b \over d}\right)} = {ad \over bc} ``` Combining rules 3 and 4, we can multiply the denominator of the bottom fraction with the numerator of the upper fraction, which gives the combined numerator, and cancels the denominator of the lower fraction; we can then multiply the denominator of the upper fraction with the numerator of the lower fraction, to give the combined denominator and cancel the denominator of the upper fraction. ```{\left({4 \over 5}\right) \over \left({2\over1}\right)} = {0.8 \over 2} = {4 \over 10} = {4*1 \over 2*5}``` As with many rules related to exponents, writing out the exponents as multiplications makes it obvious why the rule is true ```3^3*3^2 = (3*3*3)(3*3) = 3^5``` Like the previous rule, this one can be demonstrated simply by expanding the exponents out into a series of multiplications ```(4^2)^3 = (4*4)^3 = (4*4)(4*4)(4*4) = 4*4*4*4*4*4 = 4^6 = 4^{2*3}``` Thanks to the commutative property of multiplication, any series of multiplications can be rearranged without changing its value. This means that we can take a multiplication raised to a power and rearrange the resulting series of multiplications to make two exponents ```(4*5)^3 = (4*5)(4*5)(4*5) = 4*5*4*5*4*5 = 4*4*4*5*5*5 = 4^3*5^3``` ``` a^{-n} = {1\over a^n} ``` It might seem odd to have a negative exponent (since you can't multiply something by itself a negative number of times). However, if we take a closer look at the rule ``a^na^m = a^{n+m}`` we can see that it implies that ``a^{-n}`` must equal ``{1 \over a^n}``, the multiplicative inverse or reciprocal of ``a^n``. This becomes clear looking at the ``a^{n+m}`` side of the equation from rule 11. What happens if ``m`` is negative? Obviously, this will reduce the combined value of the exponent (for example, ``2^{4-2} = 2^2``). What does this mean for the left hand side of the ``a^na^m = a^{n+m}`` equation? It means that the value of, for example, ``2^4`` must be reduced to ``2^2`` when it is multiplied by ``2^{-2}``. If, as this rule states, ``a^{-n} = {1 \over a^n}``, this works out perfectly: ``2^4 * 2^{-2} = 2^4 * {1 \over 2^2} = 16 * {1 \over 4} = 4 = 2^2 = 2^{4-2}`` ```2^{-2} = {1\over2^2} =

{1\over 4}``` ``` \left({a \over b}\right)^{-n} = \left({b\over a}\right)^n ``` The reciprocal of a fraction is the fraction turned on its head: the reciprocal of ``{2 \over 3}`` is ``{3 \over 2}``. We know from the previous rule that ``a^{-n}`` is the reciprocal of ``a^n``, so we can simply convert the fraction to its reciprocal by exchanging the numerator and denominator, and then the exponent becomes positive. Positivity is such a nice thing! ```\left({1 \over 2} \right)^{-2} = {1 \over (1/2)*(1/2)} = {1 \over 1/4} = 4 = 2^2 = \left({2\over1} \right)^2``` ``` \left({a \over b}\right)^{n} = {a^n\over b^n} ``` This looks weird at first, but the reasons behind it are pretty simple. If we were paying attention when someone told us how to multiply fractions (this is doubtful, but we'll continue anyhow) we will remember that to multiply two fractions you simply multiply the numerators with each other and multiply the denominators with each other to get the resulting fraction. This rule follows from that fact. ```\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = {3*3\over4*4} = {3^2 \over 4^2}``` ``` {a^n\over a^m} = a^{n-m} ``` This one is very simple. Since division is the inverse of multiplication, multiplying a number by itself a few times and then dividing it by itself multiplied a few time is the same as just multiplying it by itself a few less times. ```{4^4\over4^2} = {4*4*4*4 \over 4*4} = {256 \over 16} = { 16 \over 1} = 16 = 4^2 = 4^{4-2} ``` This rule may seem arbitrary, but it is necessary in order to maintain consistency with other properties of exponents. Consider the rule ``a^na^m = a^{n+m}``. What happens if ``m = 0``? The right hand side of the equation will be ``a^{n+0}``, or ``a^n``. This means that in the left hand side, ``a^n`` has to be multiplied by the value of ``a^0``, but remain unchanged. The only way for this to be the case is if ``a^0 = 1``. (For some discussion of the peculiar case of ``0^0`` and why it should (probably) equal ``1``, see this article.) ```123^0 = 1 = \pi^0 = 1 = (everything)^0 = 1``` ``` a^{1 \over n} = \sqrt[n]{a} ``` As in some of the exponent properties, this rule is not an intuitive extension of the typical meaning of an exponent. Nevertheless, it fits with the all-important exponent rule ``a^na^m = a^{n+m}``. ```16^{{1\over2}+{1\over4}} = 16^{{3\over4}} = \left(16^{1\over4}\right)^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 16^{1\over4}*16^{1\over2} = \sqrt[4]{16}*\sqrt[2]{16} = 2*4 = 8``` ``` \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} ``` If ``x = \sqrt{a}`` and ``y = \sqrt{b}`` then ``\sqrt{ab} = \sqrt{x^2*y^2}`` If we write out the multiplication, this turns into ``\sqrt{x*x*y*y}``. Thanks to the commutative property of multiplication, we can rearrange the Xs and Ys and get ``\sqrt{x*y*x*y} = \sqrt{(x*y)(x*y)} = \sqrt{(x*y)^2} = x*y = \sqrt{a}*\sqrt{b}`` ```\sqrt{4*9} = \sqrt{36} = 6 = 2*3 = \sqrt{4}*\sqrt{9}``` ``` \sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} ``` Once again, by working backwards from the value of these two expressions we can see why they are equal. If ``\sqrt[m]{\sqrt[n]{a}} = x``, then we can construct ``a`` out of combinations of ``x`` and see how the whole equation works. To make things simple, we'll start with given values of ``m`` and ``n``. If ``\sqrt[2]{\sqrt[3]{a}} = x``, then ``x = \sqrt[2]{x*x}``, which also means that ``\sqrt[2]{\sqrt[3]{(x*x)*(x*x)*(x*x)}} = x``. So ``a = (x*x)*(x*x)*(x*x) = x^6 = x^{mn}``. And happily, ``\sqrt[mn]{x^{mn}} = x`` by definition, so we have ``\sqrt[m]{\sqrt[n]{a}} = x = \sqrt[mn]{x^{mn}}``. Our example is a specific case where ``m = 2`` and ``n = 3``, but since ``a`` will always be equal to ``x^{mn}``, the equation holds regardless of the values of ``m`` and ``n`` ```\sqrt[2]{\sqrt[3]{729}} = \sqrt[2]{9} = 3 = \sqrt[6]{729}``` ``` \sqrt[n]{a \over b} = {\sqrt[n]{a} \over \sqrt[n]{b}} ``` To see how this works, we can use a similar trick to the rule above. If ``x = \sqrt{a} `` and ``y = \sqrt{b}``, then: ``\sqrt{{a \over b}} = \sqrt{{x*x \over y*y}} = \sqrt{{x \over y}*{x \over y}} = \sqrt{{x \over y}^2} = {x \over y} = {\sqrt{a} \over \sqrt{b}}`` ```\sqrt[3]{8 \over 1} = 2 = {2\over 1} = {\sqrt[3]{8} \over \sqrt[3]{1}}``` ``` \tiny\text{if n is odd:} ewline ormalsize \sqrt[n]{a^n} = a ewline \tiny\text{if n is even:} ewline ormalsize \sqrt[n]{a^n} = |a| ``` If ``a`` is a positive number, then ``\sqrt[n]{a^n}`` will always equal ``a``. However, if ``a`` is negative, then the result will be positive if ``n`` is even, but it will be negative if ``n`` is odd. This comes from the fact that multiplying a negative number an even number of times always produces a positive result; an odd number of multiplications will produce a negative result. (This has the interesting side effect that there are no real numbers that are even-numbered roots of a negative number.) `````` Positive number, even root/exponent: ```\sqrt[2]{3^2} = \sqrt{9} = 3``` Positive number, odd root/exponent: ```\sqrt[3]{3^3} = \sqrt[3]{27} = 3``` Negative number, even root/exponent: ```\sqrt[2]{-3^2} = \sqrt[2]{-3*-3} = \sqrt[2]{9} = 3``` Negative number, odd root/exponent: ```\sqrt[3]{-3^3} = \sqrt[3]{-3*-3*-3} = \sqrt[3]{-27} = -3``` Algebra rules is a project by two of the folks who run The Autodidacts. A couple of autodidact math enthusiasts, we were looking for all the rules of basic algebra concisely presented in one place. We couldn’t find such a place, so we made Algebrarules.com These simple rules — applied with a pinch of imagination and a dash of arithmetic — can divide, conquer, and solve just about any practical algebra problem. If you find errata in the math, bugs in the code of Algebrarules.com, or just want to say Eh, please send us a letter or join us on our roost: @rulesofalgebra. You've made it all the way to the end! If you found this site useful, pay it forward by helping us help more people learn algebra! Enjoy sharper detail, more accurate color, lifelike lighting, believable backgrounds, and more with our new model update. Your generated images will be more polished than ever.See What's NewExplore how consumers want to see climate stories told today, and what that means for your visuals.Download Our Latest VisualGPS ReportData-backed trends. Generative AI demos. Answers to your usage rights questions. 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Our original video podcast covers it all—now on demand.Watch Now X Howdy! Here are a few very handy rules of algebra. These basic rules are useful for everything from figuring out your gas mileage to acing your next math test — or even solving equations from the far reaches of theoretical physics. Happy calculating! ```\left({a \over b}\right)^{-n} = \left({b\over a}\right)^n``` Description:The reciprocal of a fraction is the fraction turned on its head: the reciprocal of ``{2 \over 3}`` is ``{3 \over 2}``. We know from the previous rule that ``a^{-n}`` is the reciprocal of ``a^n``, so we can simply convert the fraction to its reciprocal by exchanging the numerator and denominator, and then the exponent becomes positive. Positivity is such a nice thing! ```\left({1 \over 2} \right)^{-2} = {1 \over (1/2)*(1/2)} = {1 \over 1/4} = 4 = 2^2 = \left({2\over1} \right)^2``` « Previous Rule Next Rule » Algebra rules is a project by two of the folks who run The Autodidacts. A couple of autodidact math enthusiasts, we were looking for all the rules of basic algebra concisely presented in one place. We couldn’t find such a place, so we made Algebrarules.com These simple rules — applied with a pinch of imagination and a dash of arithmetic — can divide, conquer, and solve just about any practical algebra problem. If you find errata in the math, bugs in the code of Algebrarules.com, or just want to say Eh, please send us a letter or join us on our roost: @rulesofalgebra. You've made it all the way to the end! If you found this site useful, pay it forward by helping us help more people learn algebra! S k i l l i n A L G E B R A Table of Contents | Home 6 OF The rule of symmetry The commutative rules The inverse of adding Two rules for equations ALGEBRA, we can say, is a body of formal rules. They are rules that show how something written one form may be rewritten in another form. For what is a calculation if not replacing one set of symbols into another? In arithmetic we replace '2 + 2' with '4.' In algebra, we may replace 'a + (−b)' with 'a − b.' a + (−b) = a − b. We call that a formal rule. The = sign means "may be rewritten as" or "may be replaced by." Here are some of the basic rules of algebra: 1· a = a. (1 times any number does not change it. Therefore 1 is called the identity of multiplication.) (−1)a = −a. −(−a) = a. (Lesson 2) a + (−b) = a − b. (Lesson 3) a − (−b) = a + b. (Lesson 3) Associated with these -- and with any rule -- is the rule of symmetry: For one thing, this means that a rule of algebra goes both ways. Since we may write p + (−q) = p − q -- that is, in a calculation we may replace p + (−q) with p − q -- then, symmetrically: p − q = p + (−q). We may replace p − q with p + (−q). The rule of symmetry also means that in any equation, we may exchange the sides. If 15 = 2x + 7, then we are allowed to write 2x + 7 = 15. And so the rules of algebra tell us what we are allowed to write. They tell us what is legal. Problem 1. Use the rule of symmetry to rewrite each of the following. And note that the symmetric version is also a rule of algebra. To see the answer, pass your mouse over the colored area. To cover the answer again, click "Refresh" ("Reload").Do the problem yourself first! a) 1· x = x x = 1· x b) (−1)x = −x −x = (−1)x c) x + 0 = x x = x + 0 d) 10 = 3x + 1 3x + 1 = 10 e) xy = axay axay = xy f) x + (−y) = x − y x − y = x + (−y) g) a2 + b2 = a + b 2 a + b 2 = a2 + b2 The commutative rules The order of terms does not matter. We express this in algebra by writing That is called the commutative rule of addition. It will apply to any number of terms. a + b − c + d = b + d + a − c = −c + a + d + b. The order does not matter. Example 1. Apply the commutative rule to p − q. Solution. The commutative rule for addition is stated for the operation + . Here, though, we have the operation − . But we can write p − q = p + (−q). Therefore, p − q = −q + p. * Here is the commutative rule of multiplication: The order of factors does not matter. abcd = dbac = cdba. The rule applies to any number of factors. What is more, we may associate factors in any way: (abc)d = b(dac) = (ca)(db). And so on. Example 2. Multiply 2x· 3y· 5z. Solution. The problem means: Multiply the numbers, and rewrite the letters. 2x· 3y· 5z = 2· 3· 5xyz = 30xyz. It is the style in algebra to write the numerical factor to the left of the literal factors. Problem 2. Multiply. a) 3x· 5y = 15xy b) 7p· 6q = 42pq c) 3a· 4b· 5c = 60abc Problem 3. Rewrite each expression by applying a commutative rule. a) −p + q = q + (−p) = q − p b) (−1)6 = 6(−1) c) (x − 2) + (x + 1) = (x + 1) + (x − 2) d) (x − 2)(x + 1) = (x + 1)(x − 2) Zero We have seen the following rule for 0 (Lesson 3): For any number a: 0 added to any number does not change the number. 0 is therefore called the identity of addition. The inverse of adding The inverse of an operation undoes that operation. If we start with 5, for example, and then add 4, 5 + 4, then to undo that -- to get back to 5 -- we must add −4: 5 + 4 + (−4) = 5 + 0 = 5. Adding −4 is the inverse of adding 4, and vice-versa. We say that −4 is the additive inverse of 4. In general, corresponding to every number a there is a unique number −a, such that A number combined with its inverse gives the identity. We have seen that that rule is essentially the definition of −a. Thus, the additive inverse of a is −a. And the additive inverse of −a is a. −(−a) = a. Problem 4. Transform each of the following according to a rule of algebra. a) xyz + 0 = xyz b) 0 + (−q) = −q c) −¼ + 0 = −¼ d) ½ + (−½) = 0 e) −pqr + pqr = 0 f) x + abc −abc = x g) sin x + cos x + (−cos x) = sin x The student might think that this is trigonometry, but it is not. It is g) algebra Problem 5 . Complete the following. a) pq + (−pq) = 0 b) z + (−z) = 0 c) −&2$ + &2$ = 0 d) ½x + 0 = ½x e) 0 + (−qr) = −qr f) −π + 0 = −π g) tan x + cot x + (−cot x) = tan x. Two rules for equations An equation is a statement that two things -- the two sides -- are equal. Inherent in the meaning of equal is the fact that, as long as we do the same thing to both, they will still be equal. That is expressed in the following two rules. Rule 1. If a = b, then a + c = b + c. The rule means: We may add the same number to both sides of an equation. This is the algebraic version of the axiom of arithmetic and geometry: If equals are added to equals, the sums are equal. Example 3. If x − 2 = 6, then x = 6 + 2 = 8 -- upon adding 2 to both sides. Example 4. If x + 2 = 6, then x = 6 − 2 = 4 -- upon subtracting 2 from both sides. But the rule is stated in terms of addition. Why may we subtract? Because subtraction is equivalent to addition of the negative. a − b = a + (−b). Therefore, any rule for addition is also a rule for subtraction. Note: In Example 3, adding 2 is the inverse of subtracting 2. And the effect is to transpose −2 to the other side of the equation as +2. In Example 4, the effect of subtracting 2 from both sides is to transpose +2 to the other side of the equation as −2. We will go into this more in Lesson 9. Problem 6. a) If b) If x − 1 = 5, x + 1 = 5, then then x = 6. x = 4. On adding 1 to both sides. On subtracting 1 from both sides. c) If d) If x − 4 = −6, x + 4 = −6, then then x = −2. x = −10. On adding 4 to both sides. On subtracting 4 from both sides. Rule 2. If a = b, then ca = cb. This rule means: We may multiply both sides of an equation by the same number. Example 5. If Now, what happened to 2x to make it 10x ? We multiplied it by 5. Therefore, to preserve the equality, we must multiply 3 by 5, also. 10x = 15. Example 6. If Here, we multiplied both sides by 2, and the 2's simply cancel. See Lesson 26 of Arithmetic, Example 5. Example 7. If Here, we divided both sides by 2. But the rule states that we may multiply both sides. Why may we divide? Because division is equal to multiplication by the reciprocal. In this example, we could say that we multiplied both sides by ½. Any rule for multiplication, then, is also a rule for division. Problem 7. a) If b) If x = 5, x = −7, then then 2x = 10. −4x = 28. c) If d) If x3 = 2, x4 = −2 then then x = 6. x = −8. On multiplying both sides by 3. On multiplying both sides by 4. Problem 8. Divide both sides. a) If b) If 3x = 12, −2x = 14, then then x = 4. x = −7. On dividing both sides by 3. On dividing both sides by −2. c) If d) If 6x = 5, −3x = −6, then then x = 56 x = 2. Problem 9. Changing signs on both sides. Write the line that results from multiplying both sides by −1. a) −x = 5. b) −x = −5. c) −x = 0. x = −5. x = 5. x = −0 = 0. This problem illustrates the following theorem: In any equation we may change the signs on both sides. This follows directly from the uniqueness of the additive inverse. If −a = b, then a + b = 0. We list the basic rules and properties of algebra and give examples on how they may be used. Let \(a\), \(b \) and \(c \) be real numbers, variables or algebraic expressions. 1. Commutative Property of Addition. \[a + b = b + a \] Examples: 1. real numbers \[2 + 3 = 3 + 2 \] 2. algebraic expressions \[x^2 + x = x + x^2 \] 2. Commutative Property of Multiplication. \[a \times b = b \times a \] Examples: 1. Real numbers: \[5 \times 7 = 7 \times 5 \] 2. Algebraic expressions: \[(x^3 - 2) \times x = x \times (x^3 - 2) \] 3. Associative Property of Addition. \[(a + b) + c = a + (b + c) \] Examples: 1. real numbers \[(2 + 3) + 6 = 2 + (3 + 6) \] 2. algebraic expressions \[(x^3 + 2x) + x = x^3 + (2x + x) \] 4. Associative Property of Multiplication. \[(a \times b) \times c = a \times (b \times c) \] Examples: 1. Real numbers \[(7 \times 3) \times 10 = 7 \times (3 \times 10) \] 2. Algebraic expressions \[(x^2 \times 5x) \times x = x^2 \times (5x \times x) \] 5. Distributive Properties of Addition and Multiplication. \[a \times (b + c) = a \times b + a \times c \] and \[(a + b) \times c = a \times c + b \times c \] Examples: 1. Real numbers \[2 \times (2 + 8) = 2 \times 2 + 2 \times 8 \] \[(2 + 8) \times 10 = 2 \times 10 + 8 \times 10 \] 2. Algebraic expressions \[x \times (x^4 + x) = x \times x^4 + x \times x \] \[(x^4 + x) \times x^2 = x^4 \times x^2 + x \times x^2 \] 6. The reciprocal of a non zero The reciprocal of a real number \(a\) is \[\frac{1}{a}\]. and \[a \times \left(\frac{1}{a}\right) = 1\] Examples: The reciprocal of \(5 \) is \[\frac{1}{5}\] and \[5 \times \left(\frac{1}{5}\right) = 1\]. 7. The additive inverse of a number The additive inverse of \(a \) is \[- a \] and \[a + (- a) = 0 \] Examples: additive inverse of \(-6 \) is \[-(-6) = 6 \] and \[- 6 + (6) = 0 \] 8. The additive identity The additive identity is \[0 \]. and \[a + 0 = 0 + a = a \] 9. The multiplicative identity The multiplicative identity is \[1 \]. and \[a \times 1 = 1\times a = a \] More pages related to X Howdy! Here are a few very handy rules of algebra. These basic rules are useful for everything from figuring out your gas mileage to acing your next math test — or even solving equations from the far reaches of theoretical physics. Happy calculating! a(b+c)=ab+aca(b+c) = ab+ac a(b+c)=ab+ac If you're multiplying something with a sum of two or more other values, you can distribute the multiplication to each of the values, then sum the result. ```3(4+5) = 3*4 + 3*5 = 27``` a(bc)=abca \left({b \over c} \right) = { ab \over c }a(cb)=cab A multiplication or division of the numerator of a fraction affect the fraction as a whole (and vice versa). If you need to multiply a fraction, multiplying the numerator does the job. ```6\left({1 \over 3}\right) = {(6*1) \over 3} = 2``` (ac)b=abc\left({a \over c}\right) \over b} = {a \over bc}b(ca)=bca If you divide the numerator by a particular number, it has the same effect on the fraction's overall value as if you multiply the denominator by that same number. A division above equals a multiplication below. ```{ \left({1 \over 5}\right) \over 2} = {1 \over 10} = {1 \over (2*5)}``` ``` {a \over \left({b \over c}\right)} = {ac \over b} ``` In the same fashion as the above rule, dividing the denominator of a fraction has the same effect as multiplying the numerator. A fraction below equals a multiplication above. ```{1 \over \left({6 \over 2}\right)} = {(1*2) \over 6} = {1 \over 3}``` ``` \left({a \over b}\right)+\left({c \over d}\right) = {(ad+bc) \over bd} ``` If top and bottom of a fraction are both multiplied by the same number, the fraction remains unchanged. So, we can sum two fractions by first multiplying each fraction's numerator and denominator with the other fraction's denominator. Then, since both the fractions now have the same denominator (the product of the two denominators), we can combine them into one fraction, with the sum in the numerator. ```{3 \over 5}+{1 \over 3} = {(3*3) \over (5*3)} + {(1*5) \over (3*5)} = {(1*5)+(3*3) \over (3*5)} = {14 \over 15}``` ``` \left({a \over b}\right)-\left({c \over d}\right) = {(ad-bc) \over bd} ``` This is another version of rule 5, but for subtraction of two fractions, rather than addition. ```{3 \over 5}-{1 \over 3} = {(3*3) \over (5*3)} - {(1*5) \over (3*5)} = {(3*3)-(1*5) \over (3*5)} = {9-5 \over 15} = {4 \over 15}``` ``` {(a-b) \over (c-d)} = {(b-a) \over (d-c)} ``` Reversing a subtraction gives the inverse result: ``5-3 = 2; 3-5 = -2``. In a fraction, if both the numerator and the denominator are inverted, the value of the fraction stays the same. So, if we reverse a subtraction in both the numerator and denominator, the value of the fraction is unchanged. ```{3-5 \over 2-1} = {-2 \over 1} = {5-3 \over 1-2} = {2 \over -1} = -2``` or ```{-3 \over -4} = 0.75 = {3 \over 4}``` ``` {(a+b) \over c} = {a \over c} + {b \over c} ``` Two fractions with common denominators can be added by adding the numerators and leaving the denominator unchanged. Going the other direction, we can also break apart a fraction with an addition in the numerator into two fractions (each with the common denominator). ```{(1+2) \over 4} = {3 \over 4} = {1 \over 4} + {2 \over 4}``` ``` {ac+bc \over c} = a+b ``` Division is the inverse of multiplication: if ``{a \over b} = c`` then ``b*c = a``. This means that the fraction ``{ac \over c}`` is equal to ``a``, since we are multiplying ``a`` by ``c`` and then immediately dividing it by ``c`` again, which puts us right back where we started. Since we know that ``{ac+bc \over c} = {ac \over c} + {bc \over c}`` (see rule 8), and based on the above we can see that ``{ac \over c} = a`` and ``{bc \over c} = b``, we have our result: ``a+b``. ```{(4*5)+(2*5) \over 5} = {4*5 \over 5}+{2*5 \over 5} = 4+2``` ``` {\left({a \over c}\right) \over \left({b \over d}\right)} = {ad \over bc} ``` Combining rules 3 and 4, we can multiply the denominator of the bottom fraction with the numerator of the upper fraction, which gives the combined numerator, and cancels the denominator of the lower fraction; we can then multiply the denominator of the upper fraction with the numerator of the lower fraction, to give the combined denominator and cancel the denominator of the upper fraction. ```{\left({4 \over 5}\right) \over \left({2\over1}\right)} = {0.8 \over 2} = {4 \over 10} = {4*1 \over 2*5}``` As with many rules related to exponents, writing out the exponents as multiplications makes it obvious why the rule is true ```3^3*3^2 = (3*3*3)(3*3) = 3^5``` Like the previous rule, this one can be demonstrated simply by expanding the exponents out into a series of multiplications ```(4^2)^3 = (4*4)^3 = (4*4)(4*4)(4*4) = 4*4*4*4*4*4 = 4^6 = 4^{2*3}``` Thanks to the commutative property of multiplication, any series of multiplications can be rearranged without changing its value. This means that we can take a multiplication raised to a power and rearrange the resulting series of multiplications to make two exponents ```(4*5)^3 = (4*5)(4*5)(4*5) = 4*5*4*5*4*5 = 4*4*4*5*5*5 = 4^3*5^3``` ``` a^{-n} = {1\over a^n} ``` It might seem odd to have a negative exponent (since you can't multiply something by itself a negative number of times). However, if we take a closer look at the rule ``a^na^m = a^{n+m}`` we can see that it implies that ``a^{-n}`` must equal ``{1 \over a^n}``, the multiplicative inverse or reciprocal of ``a^n``. This becomes clear looking at the ``a^{n+m}`` side of the equation from rule 11. What happens if ``m`` is negative? Obviously, this will reduce the combined value of the exponent (for example, ``2^{4-2} = 2^2``). What does this mean for the left hand side of the ``a^na^m = a^{n+m}`` equation? It means that the value of, for example, ``2^4`` must be reduced to ``2^2`` when it is multiplied by ``2^{-2}``. If, as this rule states, ``a^{-n} = {1 \over a^n}``, this works out perfectly: ``2^4 * 2^{-2} = 2^4 * {1 \over 2^2} = 16 * {1 \over 4} = 4 = 2^2 = 2^{4-2}`` ```2^{-2} = {1\over2^2} = {1\over 4}``` ``` \left({a \over b}\right)^{-n} = \left({b\over a}\right)^n ``` The reciprocal of a fraction is the fraction turned on its head: the reciprocal of ``{2 \over 3}`` is ``{3 \over 2}``. We know from the previous rule that ``a^{-n}`` is the reciprocal of ``a^n``, so we can simply convert the fraction to its reciprocal by exchanging the numerator and denominator, and then the exponent becomes positive. Positivity is such a nice thing! ```\left({1 \over 2} \right)^{-2} = {1 \over (1/2)*(1/2)} = {1 \over 1/4} = 4 = 2^2 = \left({2\over1} \right)^2``` ``` \left({a \over b}\right)^{n} = {a^n\over b^n} ``` This looks weird at first, but the reasons behind it are pretty simple. If we were paying attention when someone told us how to multiply fractions (this is doubtful, but we'll continue anyhow) we will remember that to multiply two fractions you simply multiply the numerators with each other and multiply the denominators with each other to get the resulting fraction. This rule follows from that fact. ```\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = {3*3\over4*4} = {3^2 \over 4^2}``` ``` {a^n\over a^m} = a^{n-m} ``` This one is very simple. Since division is the inverse of multiplication, multiplying a number by itself a few times and then dividing it by itself multiplied a few time is the same as just multiplying it by itself a few less times. ```{4^4\over4^2} = {4*4*4*4 \over 4*4} = {256 \over 16} = { 16 \over 1} = 16 = 4^2 = 4^{4-2} ``` This rule may seem arbitrary, but it is necessary in order to maintain consistency with other properties of exponents. Consider the rule ``a^na^m = a^{n+m}``. What happens if ``m = 0``? The right hand side of the equation will be ``a^{n+0}``, or ``a^n``. This means that in the left hand side, ``a^n`` has to be multiplied by the value of ``a^0``, but remain unchanged. The only way for this to be the case is if ``a^0 = 1``. (For some discussion of the peculiar case of ``0^0`` and why it should (probably) equal ``1``, see this article.) ```123^0 = 1 = \pi^0 = 1 = (everything)^0 = 1``` ``` a^{1 \over n} = \sqrt[n]{a} ``` As in some of the exponent properties, this rule is not an intuitive extension of the typical meaning of an exponent. Nevertheless, it fits with the all-important exponent rule ``a^na^m = a^{n+m}``. ```16^{{1\over2}+{1\over4}} = 16^{{3\over4}} = \left(16^{1\over4}\right)^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 16^{1\over4}*16^{1\over2} = \sqrt[4]{16}*\sqrt[2]{16} = 2*4 = 8``` ``` \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} ``` If ``x = \sqrt{a}`` and ``y = \sqrt{b}`` then ``\sqrt{ab} = \sqrt{x^2*y^2}`` If we write out the multiplication, this turns into ``\sqrt{x*x*y*y}``. Thanks to the commutative property of multiplication, we can rearrange the Xs and Ys and get ``\sqrt{x*y*x*y} = \sqrt{(x*y)(x*y)} = \sqrt{(x*y)^2} = x*y = \sqrt{a}*\sqrt{b}`` ```\sqrt{4*9} = \sqrt{36} = 6 = 2*3 = \sqrt{4}*\sqrt{9}``` ``` \sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} ``` Once again, by working backwards from the value of these two expressions we can see why they are equal. If ``\sqrt[m]{\sqrt[n]{a}} = x``, then we can construct ``a`` out of combinations of ``x`` and see how the whole equation works. To make things simple, we'll start with given values of ``m`` and ``n``. If ``\sqrt[2]{\sqrt[3]{a}} = x``, then ``x = \sqrt[2]{x*x}``, which also means that ``\sqrt[2]{\sqrt[3]{(x*x)*(x*x)*(x*x)}} = x``. So ``a = (x*x)*(x*x)*(x*x) = x^6 = x^{mn}``. And happily, ``\sqrt[mn]{x^{mn}} = x`` by definition, so we have ``\sqrt[m]{\sqrt[n]{a}} = x = \sqrt[mn]{x^{mn}}``. Our example is a specific case where ``m = 2`` and ``n = 3``, but since ``a`` will always be equal to ``x^{mn}``, the equation holds regardless of the values of ``m`` and ``n`` ```\sqrt[2]{\sqrt[3]{729}} = \sqrt[2]{9} = 3 = \sqrt[6]{729}``` ``` \sqrt[n]{a \over b} = {\sqrt[n]{a} \over \sqrt[n]{b}} ``` To see how this works, we can use a similar trick to the rule above. If ``x = \sqrt{a} `` and ``y = \sqrt{b}``, then: ``\sqrt{{a \over b}} = \sqrt{{x*x \over y*y}} = \sqrt{{x \over y}*{x \over y}} = \sqrt{{x \over y}^2} = {x \over y} = {\sqrt{a} \over \sqrt{b}}`` ```\sqrt[3]{8 \over 1} = 2 = {2\over 1} = {\sqrt[3]{8} \over \sqrt[3]{1}}``` ``` \tiny\text{if n is odd:} ewline ormalsize \sqrt[n]{a^n} = a ewline \tiny\text{if n is even:} ewline ormalsize \sqrt[n]{a^n} = |a| ``` If ``a`` is a positive number, then ``\sqrt[n]{a^n}`` will always equal ``a``. However, if ``a`` is negative, then the result will be positive if ``n`` is even, but it will be negative if ``n`` is odd. This comes from the fact that multiplying a negative number an even number of times always produces a positive result; an odd number of multiplications will produce a negative result. (This has the interesting side effect that there are no real numbers that are even-numbered roots of a negative number.) `````` Positive number, even root/exponent: ```\sqrt[2]{3^2} = \sqrt{9} = 3``` Positive number, odd root/exponent: ```\sqrt[3]{3^3} = \sqrt[3]{27} = 3``` Negative number, even root/exponent: ```\sqrt[2]{-3^2} = \sqrt[2]{-3*-3} = \sqrt[2]{9} = 3``` Negative number, odd root/exponent: ```\sqrt[3]{-3^3} = \sqrt[3]{-3*-3*-3} = \sqrt[3]{-27} = -3``` Algebra rules is a project by two of the folks who run The Autodidacts. A couple of autodidact math enthusiasts, we were looking for all the rules of basic algebra concisely presented in one place. We couldn’t find such a place, so we made Algebrarules.com These simple rules — applied with a pinch of imagination and a dash of arithmetic — can divide, conquer, and solve just about any practical algebra problem. If you find errata in the math, bugs in the code of Algebrarules.com, or just want to say Eh, please send us a letter or join us on our roost: @rulesofalgebra. You've made it all the way to the end! If you found this site useful, pay it forward by helping us help more people learn algebra!