Combinatorics problems

Combinatorics Problem Given two finite sets A and B, with |A|=m and |B|=n, find the number of distinct functions (mappings) from set A to set B. Using the multiplication principle, we can determine that there are n^m possible mappings. **One-to-One Functions** Find the number of distinct one-to-one functions (mappings) from set A to set B. Using the multiplication principle, we can determine that there are P^n_m possible one-to-one functions. **Binomial Experiment** Given an urn containing 30 red balls and 70 green balls, find the probability of getting exactly k red balls in a sample of size 20 if sampling is done with replacement. This is equivalent to the binomial experiment, so we can use the binomial formula: P(k red balls) = {20 choose k} (0.3)^k(0.7)^(20-k). **Without Replacement** Given an urn containing 30 red balls and 70 green balls, find the probability of getting exactly k red balls in a sample of size 20 if sampling is done without replacement. To solve this problem, we need to count the number of ways to choose k red balls and (20-k) green balls using the multiplication principle: |A| = {30 choose k}{70 choose (20-k)}. Then, P(A) = |A| / |S|. **Conditional Probability** Given that there are k people in a room with probabilities p(k=5)=1/4, p(k=10)=1/4, and p(k=15)=1/2, find the probability that at least two of them have been born in the same month. Since all months are equally likely, we can assume that each person's birth month is independent. To solve this problem, we need to consider the possible cases where at least two people share a birth month and calculate their probabilities using conditional probability. Note: The original text has multiple problems, but I've only paraphrased the first three problems above. Problem 1: Find the probability that at least two people out of k people have birthdays in the same month, given that there are 12 months. Solution: The problem can be solved using the inclusion-exclusion principle. Let A_k be the event that at least two people out of k people have birthdays in the same month. Then we need to find $P(A_k)$ for $k \in \{2,3,4,...,12\}$. The solution involves calculating the probability using the formula $P(A_k)=1-\frac{P^{12}_k}{12^k}$. Problem 2: Find the probability that at least one person receives their own hat when N guests are randomly assigned hats. Solution: Let A_i be the event that the ith person receives their own hat. Then we need to find $P(E)$, where $E=A_1 \cup A_2 \cup ... \cup A_N$. The solution involves using the inclusion-exclusion principle and symmetry to calculate the probability. Problem 3: Find the number of distinct solutions to the equation $x_1+x_2+...+x_n=k$, where x_i is a non-negative integer, under certain restrictions. Solution: The problem can be solved by converting the restrictions to match the general form of the equation. Let $y_i=x_i-1$ and $z_j=x_{n+1-j}-1$. Then we need to find the number of solutions to the equation $y_1+y_2+...+y_n+z_1+z_2+...+z_n=97$, where y_i and z_j are non-negative integers. The solution involves using combinatorial formulas, specifically ${n+k-1 \choose k}={n+k-1 \choose n-1}$. Problem 4: (The matching problem) Find the probability that at least one person receives their own hat when N guests are randomly assigned hats. Solution: This is a variant of the previous problem. The solution involves using the inclusion-exclusion principle and symmetry to calculate the probability, similar to the second part of Problem 2. To find the probability P(E) that at least one student chooses the first bedroom in the four-bedroom house, we can use the principle of inclusion-exclusion. We need to calculate the probabilities of different combinations of students choosing rooms. First, we find the probability P(A1) that a single student chooses a double room, which is (N-1)!/N!. Then, we find the probability P(A1 ∩ A2) that two students choose double rooms, and so on. We can use these probabilities to calculate the desired probability using equation 2.5. By simplifying the expression, we obtain an alternating series of fractions, which approaches a limit as N becomes large. Interestingly, this limit is 1 - 1/e, where e is the base of the natural logarithm. The final paragraph provides two more problems related to combinatorics and probability. The first problem involves five roommates choosing bedrooms in a house with four bedrooms, while the second problem deals with a restaurant owner tracking customer purchases. This collection of problems has been posted by Orlando Doehring and includes various mathematical challenges from different competitions and events. The problems are categorized into several sections, including IMO (International Mathematical Olympiad), ISL and ILL, other competitions, and specific problems related to team selection tests for countries like China and Vietnam. Some notable problems in this collection include: 1. A problem about partitioning a set of consecutive numbers into two subsets such that the product of each subset is equal. 2. A problem involving 100 coplanar points and requiring proof that at most 70% of the triangles formed by these points have all angles acute. 3. A problem about finding a set of points in the plane such that any point on the set has exactly m other points at unit distance. 4. A problem about dividing a set of ten distinct two-digit numbers into disjoint subsets with the same sum. 5. A problem about determining if it's possible to draw 1975 distinct points on a circle of radius 1 such that the distance between any two points is a rational number. The collection also includes problems related to the packing of cubes in a box, the properties of integers and prime divisors, and the behavior of certain mathematical operations applied to assigned numbers at the vertices of a regular pentagon. The problem involves finding maximal lengths of words under certain conditions, as well as solving combinatorial problems related to sequences, game theory, and cryptography. For instance, Problem 22 asks for a sequence of integer numbers with specific properties, while Problem 23 deals with the distribution of scores among three players. Additionally, Problems 24 and 25 focus on the number of combinations required to open a safe or find a particular natural number, respectively. Finally, Problems 26 and 27 involve proving statements related to circular permutations and lattice points, while Problem 28 explores a scenario involving a balance with bags of coins. Find a way to do this and explain it. An n × n chessboard with every number from 1 to n^2 is numbered. Prove that for any two neighboring squares with common edges, their numbers differ by at least n. Determine for which positive integers k the set X = {1990, 1990 + 1, 1990 + 2, ..., 1990 + k} can be partitioned into two disjoint subsets A and B such that the sum of the elements in A is equal to the sum of the elements in B. Let n, k ∈ Z+ with k ≤ n. Let S be a set containing n distinct real numbers. Let T be a set of all real numbers of the form x1 + x2 + ... + xk where x1, x2, ..., xk are distinct elements from S. Prove that T contains at least k(n − k) + 1 distinct elements. In a city where age is measured in real numbers, citizens either know each other or do not. If they do not, there exists a chain of citizens connecting them. Each female citizen provides the average ages of those she knows. Prove that this information uniquely determines the ages of all female citizens. At a meeting of 12k people, each person greets exactly 3k+6 others. The number who exchange greetings with both is the same for any two people. How many people are at the meeting? We are given r and a rectangular board ABCD with dimensions AB = 20, BC = 12. Divide the rectangle into a grid of 20 × 12 unit squares. Show that the task cannot be done when r is divisible by 2 or 3. Prove that the task is possible when r = 73. Can the task be done when r = 97? Determine whether there exist two disjoint infinite sets A and B of points in the plane satisfying certain conditions. The problem requires solving six combinatorics problems from various International Mathematics Olympiads. 1. A rectangular array of numbers has an integer sum in each row and column, but some numbers are not integers. The task is to prove that these non-integers can be rounded up or down without changing the row-sums and column-sums. 2. In a contest with m candidates and n judges (n being odd), each judge agrees on at most k candidates. The problem asks to prove that km ≥ (n-1)/2n. 3. Given two sets A and B of N residues mod N2, the task is to show that A + B = {a + b | a ∈ A, b ∈ B} contains at least half of all residues mod N2. 4. Every integer is given one of four colors, red, blue, green or yellow. Two odd integers x and y are such that |x| ≠ |y|. The problem asks to show that there are two integers of the same color whose difference has a certain value (x, y, x+y or x-y). 5. Given n points in the plane with specific properties, the task is to prove that there exists an integer f(n) such that if m(S) = f(n), then S forms a convex polygon. 6. Pawns are placed on squares of an n × n chessboard such that no column or row contains k adjacent unoccupied squares. The problem asks to find the least number m for which this is possible. 7. Given a sequence A of 2001 positive integers, the task is to find the greatest value of m for which there exist three subsequences (ai, aj, ak) with specific properties. 8. For an odd integer n and sets c1, c2, ..., cn, the problem asks to prove that there exist permutations a and b of {1, 2, ..., n} such that n! divides S(a)-S(b). 9. A party has people who form k-cliques (sets of k acquainted individuals). The task is to prove that if certain conditions are met, then only two or fewer people can leave without forming any remaining clique. 10. Finally, the problem asks to show that points in a plane are colored red or blue under specific conditions and relates it to graph theory. Note: Each problem statement has been paraphrased to make them more concise and easier to understand while maintaining their original essence. The problem set includes various mathematical problems from the China TST Olympiad, covering topics such as graph theory, geometry, and combinatorics. Problem 57 asks to find the maximum number of circles that can be drawn with arbitrary 7 points in the plane, where each point is connected by a circle through every 4 possible concyclic points. Problem 58 requires finding the number of "good" circles formed by 5 points in a plane, where three points determine a circle with one inside and one outside another point. Problem 59 asks to prove that there exist two vertices on a polyhedron such that the sum of excen-tricity at those vertices is less than or equal to 4. Problem 60 deals with finding the maximum number of multiple-choice problems for 16 students, given that any two students have at most one answer in common. Problem 61 involves proving that deleting any square from a specific table results in a shape that can be cut into pieces of a particular form. Problem 62 asks to prove that there exists a graph with n colors and no triangles, which is known as an "n-colored" graph. Problem 63 has two sub-problems: finding the smallest number of squares required to obtain a key (a set of squares that can determine the entire grid) in an n-code; and finding the smallest number of squares along the diagonals required to form a key in an n-code. Problem 64 involves finding the smallest n such that if 5 vertices of a regular n-gon are colored red, there exists a line of symmetry where every red point is reflected across it to a non-red point. Problem 65 requires determining the minimum number of people who could have correctly answered a question with the most common correct answer among a group of people who took a test with multiple-choice questions. Problem 66 asks to find the maximum number of elements in set S such that the distance between any two elements is at least 5, where each element is an 8-element sequence of 0s and 1s. Finally, Problem 67 involves determining the minimum number of representatives required from each country (A, B, C) to ensure that at least one representative can win every round of competition. The problem describes a scenario where monkeys receive peanuts in a specific pattern. The first peanut is given to the leader of the monkeys, followed by skips of increasing size (2, 4, 6, etc.). The number of peanuts skipped before giving out the next one increases with each subsequent peanut. The problem also includes four other problems: 1. How many monkeys don't receive peanuts? 2. How many edges on a polygon have both monkeys at its vertex receiving peanuts? Additionally, there are three more problems involving club members and a space with orthogonal coordinate system. Let me know if you'd like me to paraphrase any of these problems in particular! Given problem statement text here The task is to find the maximum number of clowns, denoted as n, that can be accommodated in a circus while adhering to specific constraints regarding their assigned colors. The key requirements include using at least five different colors and ensuring that no two clowns have identical sets of colors and no more than 20 clowns are associated with any given color. Three edges are colored blue. Ninety-three (China West Mathematical Olympiad 2003, Problem 8). One thousand six hundred fifty students are arranged in twenty-two rows and seventy-five columns. It is known that in any two columns, the number of pairs of students in the same row and of the same sex is not greater than eleven. Prove that the number of boys is not greater than nine hundred twenty-eight. Ninety-four (China Girls Mathematical Olympiad 2005, Problem 4). Determine all positive real numbers a such that there exists a positive integer n and sets A1, A2, ..., An satisfying the following conditions: • every set Ai has infinitely many elements; • every pair of distinct sets Ai and Aj do not share any common element • the union of sets A1, A2, ..., An is the set of all integers; • for every set Ai, the positive difference of any pair of elements in Ai is at least a. Fourteen fifteen. Ninety-five (Germany Bundeswettbewerb Mathematik 2007, Round 2, Problem 2). At the start of the game there are r red and g green pieces/stones on the table. Hojoo and Kestutis make moves in turn. Hojoo starts. The person due to make a move chooses a color and removes k pieces of this color. The number k has to be a divisor of the current number of stones of the other color. The person removing the last piece wins. Who can force the victory? Ninety-six (Germany Bundeswettbewerb Mathematik 2008, Round 1, Problem 1). Fedja used matches to put down the equally long sides of a parallelogram whose vertices are not on a common line. He figures out that exactly seven or nine matches, respectively, fit into the diagonals. How many matches compose the parallelogram's perimeter? Ninety-seven (Tuymaada 2008, Senior League, First Day, Problem 1). Several irrational numbers are written on a blackboard. It is known that for every two numbers a and b on the blackboard, at least one of the numbers a b+1 and b a+1 is rational. What maximum number of irrational numbers can be on the blackboard? Ninety-eight (Tuymaada 2008, Senior League, Second Day, Problem 5). Every street in the city of Hamiltonville connects two squares, and every square may be reached by streets from every other. The governor discovered that if he closed all squares of any route not passing any square more than once, every remained square would be reachable from each other. Prove that there exists a circular route passing every square of the city exactly once. Ninety-nine (Tuymaada 2008, Junior League, First Day, Problem 3). One hundred unit squares of an infinite squared plane form a ten by ten square. Unit segments forming these squares are colored in several colors. It is known that the border of every square with sides on grid lines contains segments of at most two colors. (Such square is not necessarily contained in the original ten by ten square.) What maximum number of colors may appear in this coloring? One hundred (Iran PPCE 1997, Exam 2, Problem 1). Let k, m, n be integers such that one n ≤ m − 1 ≤ k. Determine the maximum size of a subset S of the set {1, 2, 3, ..., k − 1, k} such that no n distinct elements of S add up to m. Fifteen "102 Combinatorial Problems" consists of carefully selected problems that have been used in the training and testing of the USA International Mathematical Olympiad (IMO) team. Key features: * Provides in-depth enrichment in the important areas of combinatorics by reorganizing and enhancing problem-solving tactics and strategies * Topics include: combinatorial arguments and identities, generating functions, The book covers a wide range of topics including recursive formulas, numerical calculations, probability theory, and algebraic concepts such as polynomials and equations. It also delves into complex numbers in geometric contexts, algorithmic reasoning, advanced geometric ideas, and functional equations with classical inequality solutions. Organized systematically, the book develops students' combinatorial skills gradually while expanding their understanding of mathematics as a whole. Beyond its utility for students preparing for math competitions or teachers seeking to hone their own skills, this resource serves as a valuable addition to any mathematical library, sparking curiosity in related areas beyond just combinatorics itself. Featuring 102 engaging combinatorial problems for students to tackle and explore.

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