

Continue





























x
¯




{\displaystyle \left(\prod \_{i=1}^{n}x\_{i}\right)^{\frac {1}{n}}=\left(x\_{1}x\_{2}\cdots x\_{n}\right)^{\frac {1}{n}}}

For example, the geometric mean of four values: 4, 36, 45, 50, 75 is: 





(
4
×
36
×
45
×
50
×
75

)

1

5


=
24
300
000

5


=
30.


{\displaystyle (4\times 36\times 45\times 50\times 75)^{\frac {1}{5}}={\sqrt[{5}]{24\;300\;000}}=30.}

The harmonic mean is an average which is useful for sets of numbers which are defined in relation to some unit, as in the case of speed (i.e., distance per unit of time): 





x
¯


=
n


(

∑

i
=
1


n



1

x

i




)

−
1




{\displaystyle {\bar {x}}={n\left(\sum \_{i=1}^{n}{\frac {1}{x\_{i}}}\right)^{-1}}

For example, the harmonic mean of the five values: 4, 36, 45, 50, 75 is 





5


1


4


+


1


36


+


1


45


+


1


50


+


1


75


=
5


1


3


=
15.


{\displaystyle {\frac {5}{\left({\frac {1}{4}}\right)+\left({\frac {1}{36}}\right)+\left({\frac {1}{45}}\right)+\left({\frac {1}{50}}\right)+\left({\frac {1}{75}}\right)}}={\frac {5}{\left\{{\frac {1}{3}}\right\}}}=15.}

If we have five pumps that can empty a tank of a certain size in respectively 4, 36, 45, 50, and 75 minutes, then the harmonic mean of 15 



{\displaystyle 15}

 tells us that these five different pumps working together will pump at the same rate as much as five pumps that can each empty the tank in 15 



{\displaystyle 15}

 minutes. Proof without words of the AM–GM inequality:PR is the diameter of a circle centered on O; its radius AO is the arithmetic mean of a and b. Using the geometric mean theorem, triangle PGR's altitude GQ is the geometric mean. For any ratio a:b, AO ≥ GQ. Main article: QM-AM-GM-HM inequalities AM, GM, and HM of nonnegative real numbers satisfy these inequalities:[5] 



A
M
≥
G
M
≥
H
M


{\displaystyle \mathrm {AM} \geq \mathrm {GM} \geq \mathrm {HM} \ \}

Equality holds if all the elements of the given sample are equal. See also: Average § Statistical location Comparison of the arithmetic mean, median, and mode of two skewed (log-normal) distributions Geometric visualization of the mode, median and mean of an arbitrary probability density function[6] In descriptive statistics, the mean may be confused with the median, mode or mid-range, as any of these may incorrectly be called an "average" (more formally, a measure of central tendency). The mean of a set of observations is the arithmetic average of the values; however, for skewed distributions, the mean is not necessarily the same as the middle value (median), or the most likely value (mode). For example, mean income is typically skewed upwards by a small number of people with very large incomes, so that the majority have an income lower than the mean. By contrast, the median income is the level at which half the population is below and half is above. The mode income is the most likely income and favors the larger number of people with lower incomes. While the median and mode are often more intuitive measures for such skewed data, many skewed distributions are in fact best described by their mean, including the exponential and Poisson distributions. Main article: Expected value See also: Population mean The mean of a probability distribution is the long-run arithmetic average value of a random variable having that distribution. If the random variable is denoted by 



X


{\displaystyle X}

, then the mean is also known as the expected value of 



X


{\displaystyle X}

 (denoted 



E
(
X
)


{\displaystyle E(X)}

). For a discrete probability distribution, the mean is given by 



∑
x
P
(
x
)


{\displaystyle \textstyle \sum xP(x)}

, where the sum is taken over all possible values of the random variable and 



P
(
x
)


{\displaystyle P(x)}

 is the probability mass function. For a continuous distribution, the mean is 




f
−
∞


∞



x
f
(
x
)
d
x


{\displaystyle \textstyle \int \_{-\infty }^{\infty }xf(x)\,dx}

, where 



f
(
x
)


{\displaystyle f(x)}

 is the probability density function.[7] In all cases, including those in which the distribution is neither discrete nor continuous, the mean is the Lebesgue integral of the random variable with respect to its probability measure. The mean need not exist or be finite; for some probability distributions the mean is infinite (+∞ or −∞), while for others the mean is undefined. The generalized mean, also known as the power mean or Hölder mean, is an abstraction of the quadratic, arithmetic, geometric, and harmonic means. It is defined for a set of n positive numbers xi by 





x
¯


(
m
)


=
(


1

n



∑

i
=
1


n



x

i


m




)

1

m




{\displaystyle {\bar {x}}(m)=\left({\frac {1}{n}}\sum \_{i=1}^{n}x\_{i}^{m}\right)^{\frac {1}{m}}}

 [1] By choosing different values for the parameter m, the following types of means are obtained: lim m → ∞ 



{\displaystyle \lim \_{m\to \infty }}

 maximum of 



x

i




{\displaystyle x\_{i}}

 lim m → 2 



{\displaystyle \lim \_{m\to 2}}

 quadratic mean lim m → 1 



{\displaystyle \lim \_{m\to 1}}

 arithmetic mean lim m → 0 



{\displaystyle \lim \_{m\to 0}}

 geometric mean lim m → −1 



{\displaystyle \lim \_{m\to -1}}

 harmonic mean lim m → −∞ 



{\displaystyle \lim \_{m\to -\infty }}

 minimum of 



x

i




{\displaystyle x\_{i}}

 This can be generalized further as the generalized F-mean 





x
¯


=
f
−
1


(


1

n



∑

i
=
1


n



f
(

x

i


)


)


{\displaystyle {\bar {x}}=F^{-1}\left({\frac {1}{n}}\sum \_{i=1}^{n}\left({\frac {1}{f(x\_{i})}\right)}\right)}

 and again a suitable choice of an invertible f will give 



f
(
x
)
=

x

m




{\displaystyle f(x)=x^{m}}

 power mean, 



f
(
x
)
=
x


{\displaystyle f(x)=x}

 arithmetic mean, 



f
(
x
)
=
ln
⁡
(
x
)


{\displaystyle f(x)=\ln(x)}

 geometric mean, 



f
(
x
)
=
x
−
1


=
1

x




{\displaystyle f(x)=x^{-1}={\frac {1}{x}}}

 harmonic mean. The weighted arithmetic mean (or weighted average) is used if one wants to combine average values from different sized samples of the same population: 





x
¯


=


∑

i
=
1


n


w

i



x

i




∑

i
=
1


n


w

i




.


{\displaystyle {\bar {x}}={\frac {\sum \_{i=1}^{n}w\_{i}{\bar {x}\_{i}}}{\sum \_{i=1}^{n}w\_{i}}}.}

 [1] Where 





x
¯


i




{\displaystyle {\bar {x}\_{i}}}

 and 




w

i




{\displaystyle w\_{i}}

 are the mean and size of sample i 



{\displaystyle i}

 respectively. In other applications, they represent a measure for the reliability of the influence upon the mean by the respective values. Sometimes, a set of numbers might contain outliers (i.e., data values which are much lower or much higher than the others). Often, outliers are erroneous data caused by artifacts. In this case, one can use a truncated mean. It involves discarding given parts of the data at the top or the bottom end, typically an equal amount at each end and then taking the arithmetic mean of the remaining data. The number of values removed is indicated as a percentage of the total number of values. The interquartile mean is a specific example of a truncated mean. It is simply the arithmetic mean after removing the lowest and the highest quarter of values. 





x
¯


=


2

n



∑

i
=
4


+
1


3


4


n


x

i




{\displaystyle {\bar {x}}={\frac {2}{n}}\sum \_{i={\frac {n}{4}}+1}^{\frac {3}{4}n}\!x\_{i}}

 assuming the values have been ordered, so is simply a specific example of a weighted mean for a specific set of weights. Main article: Mean of a function In some circumstances, mathematicians may calculate a mean of an infinite (or even an uncountable) set of values. This can happen when calculating the mean value 



y
avg


{\displaystyle y\_{\text{avg}}}

 of a function 



f
(
x
)


{\displaystyle f(x)}

. Intuitively, a mean of a function can be thought of as calculating the area under a section of a curve, and then dividing by the length of that section. This can be done crudely by counting squares on graph paper, or more precisely by integration. The integration formula is written as: 



y
avg
(
a
,
b
)
=


1

b
−
a



∫

a


b


f
(
x
)
d
x


{\displaystyle y\_{\text{avg}}(a,b)={\frac {1}{b-a}}\int \limits \_{a}^{b}f(x)\,dx}

 In this case, care must be taken to make sure that the integral converges. But the mean may be finite even if the function itself tends to infinity at some points. Angles, times of day, and other cyclical quantities require modular arithmetic to add and otherwise combine numbers. These quantities can be averaged using the circular mean. In all these situations, it is possible that no mean exists, for example if all points being averaged are equidistant. Consider a color wheel—there is no mean to the set of all colors. Additionally, there may not be a unique mean for a set of values: for example, when averaging points on a clock, the mean of the locations of 11:00 and 13:00 is 12:00, but this location is equivalent to that of 00:00. The Fréchet mean gives a manner for determining the "center" of a mass distribution on a surface or, more generally, Riemannian manifold. Unlike many other means, the Fréchet mean is defined on a space whose elements cannot necessarily be added together or multiplied by scalars. It is sometimes also known as the Karcher mean (named after Hermann Karcher). In geometry, there are thousands of different definitions for the center of a triangle that can all be interpreted as the mean of a triangular set of points in the plane.[8] This is an approximation to the mean for a moderately skewed distribution.[9] It is used in hydrocarbon exploration and is defined as: 



m
=
0.3

P

10


+
0.4

P

50


+
0.3

P

90




{\displaystyle m=0.3P\_{10}+0.4P\_{50}+0.3P\_{90}}

 where 




P

10




{\textstyle P\_{10}}

, 




P

50




{\textstyle P\_{50}}

 and 




P

90




{\textstyle P\_{90}}

 are the 10th, 50th and 90th percentiles of the distribution, respectively. Main category: Means Arithmetic-geometric mean Arithmetic-harmonic mean Cesàro mean Chisini mean Contraharmonic mean Elementary symmetric mean Geometric-harmonic mean Grand mean Heinz mean Heronian mean Idetric mean Lehmer mean Logarithmic mean Moving average Neuman–Sándor mean Quasi-arithmetic mean Root mean square (quadratic mean) Rényi's entropy (a generalized F-mean) Spherical mean Stolarsky mean Weighted geometric mean Weighted harmonic mean Mathematics portal Statistical dispersion Central tendency Median Mode Descriptive statistics Kurtosis Law of averages Mean value theorem Moment (mathematics) Summary statistics Taylor's law 



ⱥ

{\displaystyle \mu }

 Pronounced "x bar" 



ⱥ

{\displaystyle \mu }

 Greek letter μ, pronounced /mjuː/. 



ⱥ

a
b
c
d


{\text{Mean|mathematics}}

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