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the magnitudes r, and add the angles, using the fact that $(\cos(x) + i \sin(x)) (\cos(y) + i \sin(y)) = \cos(x+y) + i \sin(x+y)$. BTW, this is a great way to remember the angle addition identities. If you're seeing this message, it means we're having trouble loading external resources on our website. If you're behind a web filter, please make sure that the domains [*kastatic.org](https://www.kastatic.org) and [*kasandbox.org](https://www.kasandbox.org) are unblocked. In order to continue enjoying our site, we ask that you confirm your identity as a human. Thank you very much for your cooperation. A Complex Number is a combination of a Real Number and an Imaginary Number: A Real Number is the type of number we use every day. Examples: 12, 38, $\frac{1}{2}$, 0, -2000 An Imaginary Number, when squared gives a negative result: The "unit" imaginary number when squared equals -1 $i^2 = -1$ Examples: $5i$, $-3.6i$, $i/2$, $500i$ $3.6 + 4i$ (real part is 3.6, imaginary part is 4) $-0.02 + 1.2i$ (real part is -0.02 , imaginary part is 1.2) $25 - 0.3i$ (real part is 25, imaginary part is -0.3) Either part can be zero: $0 + 2i$ (no real part, imaginary part is 2) same as $2i$ $4 + 0i$ (real part is 4, no imaginary part) same as 4 Multiplying To multiply complex numbers: Each part of the first complex number gets multiplied by each part of the second complex number Just use "FOIL", which stands for "Firsts, Outers, Inners, Lasts" (see Binomial Multiplication for more details): Firsts: $a \times c$ Outers: $a \times di$ Inners: $bi \times c$ Lasts: $bi \times di$ $(a+bi)(c+di) = ac + adi + bci + bdi^2$ Like this: $(3 + 2i)(1 + 7i) = 3 \times 1 + 3 \times 7i + 2i \times 1 + 2i \times 7i = 3 + 21i + 2i + 14i^2 = 3 + 21i + 2i - 14$ (because $i^2 = -1$) $= -11 + 23i$ Here is another example: $(1 + i)^2 = (1 + i)(1 + i) = 1 \times 1 + 1 \times i + 1 \times i + i^2 = 1 + 2i - 1$ (because $i^2 = -1$) $= 0 + 2i$ But There is a Quicker Way! Use this rule: $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$ $(3 + 2i)(1 + 7i) = (3 \times 1 - 2 \times 7) + (3 \times 7 + 2 \times 1)i = -11 + 23i$ Why Does That Rule Work? It is just the "FOIL" method after a little work: $(a+bi)(c+di) = ac + adi + bci + bdi^2$ FOIL method $= ac + adi + bci - bd$ (because $i^2=-1$) $= (ac - bd) + (ad + bc)i$ (gathering like terms) And there you have the $(ac - bd) + (ad + bc)i$ pattern. This rule is certainly faster, but if you forget it, just remember the FOIL method. Now let's see what multiplication looks like on the Complex Plane. The Complex Plane This is the complex plane: It is a plane for complex numbers! We can plot a complex number like $3 + 4i$: It is placed 3 units along (the real axis), and 4 units up (the imaginary axis). Multiplying By i Let's multiply it by i: $(3 + 4i) \times i = 3i + 4i^2$ Which simplifies to (because $i^2 = -1$): $-4 + 3i$ And here is the cool thing ... It's the same as rotating by a right angle (90° or $\pi/2$) Was that just a weird coincidence? Let's try multiplying by i again: $(-4 + 3i) \times i = -4i + 3i^2 = -3 - 4i$ and again: $(-3 - 4i) \times i = -3i - 4i^2 = 4 - 3i$ and again: $(4 - 3i) \times i = 4i - 3i^2 = 3 + 4i$ Well, isn't that stunning? Each time it rotates by a right angle, until it ends up where it started. Let's try it on the number 1: $1 \times i = i$ $i \times i = -1$ $-1 \times i = -i$ $-i \times i = 1$ Back to 1 again! Each time a right angle rotation. Choose your own complex number and try that for yourself, it is good practice. Let's look more closely at angles now. Polar Form Our friendly complex number $3 + 4i$: Here it is again, but in polar form: (distance and angle) So the complex number $3 + 4i$ can also be shown as distance (5) and angle (0.927 radians). How do we do the conversions? We can do a Cartesian to Polar conversion: $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ $\theta = \tan^{-1}(y/x) = \tan^{-1}(4/3) = 0.927$ (to 3 decimals) We can also take Polar coordinates and convert them to Cartesian coordinates: $x = r \times \cos(\theta) = 5 \times \cos(0.927) = 5 \times 0.6002... = 3$ (close enough) $y = r \times \sin(\theta) = 5 \times \sin(0.927) = 5 \times 0.7998... = 4$ (close enough) In fact, a common way to write a complex number in Polar form is $x + iy = r \cos \theta + i r \sin \theta = r(\cos \theta + i \sin \theta)$ And "cos $\theta + i \sin \theta$ " is often shortened to "cis θ ", so: $x + iy = r \text{ cis } \theta$ cis is just shorthand for $\cos \theta + i \sin \theta$ So we can write: $3 + 4i = 5 \text{ cis } 0.927$ In some subjects, like electronics, "cis" is used a lot! Now For Some More Multiplication Let's try another multiplication: $(1+i)(3+i) = 1(3+i) + i(3+i) = 3 + i + 3i + i^2 = 3 + 4i - 1 = 2 + 4i$ And here is the result on the Complex Plane: But it is more interesting to see those numbers in Polar Form: Convert $1+i$ to Polar: $r = \sqrt{1^2 + 1^2} = \sqrt{2} \theta = \tan^{-1}(1/1) = 0.785$ (to 3 decimals) Convert $3+i$ to Polar: $r = \sqrt{3^2 + 1^2} = \sqrt{10} \theta = \tan^{-1}(1/3) = 0.322$ (to 3 decimals) Convert $2+4i$ to Polar: $r = \sqrt{2^2 + 4^2} = \sqrt{20} \theta = \tan^{-1}(4/2) = 1.107$ (to 3 decimals) Have a look at the r values for a minute. Are they related somehow? And what about the θ values? Here is that multiplication in one line (using "cis"): $(\sqrt{2} \text{ cis } 0.785) \times (\sqrt{10} \text{ cis } 0.322) = \sqrt{20} \text{ cis } 1.107$ This is the interesting thing: $\sqrt{2} \times \sqrt{10} = \sqrt{20}$ $0.785 + 0.322 = 1.107$ So: The magnitudes get multiplied. And the angles get added. When multiplying in Polar Form: multiply the magnitudes, add the angles. And that is why multiplying by i rotates by a right angle: i has a magnitude of 1 and forms a right angle on the complex plane Squaring To square a complex number, multiply it by itself: multiply the magnitudes: magnitude $\times$ magnitude $=$ magnitude² add the angles: angle $+$ angle $= 2 \times$, so we double them. Result: square the magnitudes, double the angle. Example: Let us square $1 + 2i$: $(1 + 2i)(1 + 2i) = 1 + 4i + 4i^2 = -3 + 4i$ On the diagram the angle looks to be (and is!) doubled. Also: The magnitude of $(1+2i) = \sqrt{1^2 + 2^2} = \sqrt{5}$ The magnitude of $(-3+4i) = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ So the magnitude got squared, too. In general, a complex number like: $r(\cos \theta + i \sin \theta)$ When squared becomes: $r^2(\cos 2\theta + i \sin 2\theta)$ (the magnitude r gets squared and the angle θ gets doubled.) Or in the shorter "cis" notation: $(r \text{ cis } \theta)^2 = r^2 \text{ cis } 2\theta$ De Moivre's Formula And the mathematician Abraham de Moivre found it works for any integer exponent n: $[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$ (the magnitude becomes r^n the angle becomes $n\theta$.) Or in the shorter "cis" notation: $(r \text{ cis } \theta)^n = r^n \text{ cis } n\theta$ Example: What is $(1+i)^6$ Convert $1+i$ to Polar: $r = \sqrt{1^2 + 1^2} = \sqrt{2} \theta = \tan^{-1}(1/1) = \pi/4$ In "cis" notation: $1+i = \sqrt{2} \text{ cis } \pi/4$ Now, with an exponent of 6, r becomes r^6 , θ becomes 6θ : $(\sqrt{2} \text{ cis } \pi/4)^6 = (\sqrt{2})^6 \text{ cis } 6\pi/4 = 8 \text{ cis } 3\pi/2$ The magnitude is now 8, and the angle is $3\pi/2 (=270^\circ)$ Which is also $0-8i$ (see diagram) Summary Use "FOIL" to multiply complex numbers, Or use the formula: $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$ Or use polar form and then multiply the magnitudes and add the angles. De Moivre's Formula can be used for integer exponents: $[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$ Polar form $r \cos \theta + i r \sin \theta$ is often shortened to $r \text{ cis } \theta$ Copyright © 2020 MathsIsFun.com

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